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Vector Mesons in $q\bar{q}q\bar{q}$ Systems —Revised Version II—

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abstract

We discuss the vector mesons, $I^G(J^{PC}) = 1^+(1^{--})$ and $1^-(1^{-+})$ in $\mathbf{10} \oplus \overline{\mathbf{10}}$ representations, decaying into two ground-state octets. We explore the consequences of the flavor $SU(3)$ symmetry on the properties of these vector mesons. The flavor octets considered are $\{\pi\}$, $\{\rho\}$, $\{b_1\}$, $\{a_1\}$ and $\{a_2\}$, labeled by the isovector state in the representation.

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An exotic meson, the $\pi_1(1400)$ with $I^G(J^{PC}) = 1^-(1^{-+})$, has been seen to decay into a p-wave $\eta\pi$ system [1]. It has been shown [2] that, if the η meson is assumed to be a (pure) member of the pion octet and in the limit of flavor $SU(3)$ conservation in its decay, a p-wave $\eta\pi$ system belongs to a flavor $\mathbf{10} \oplus \overline{\mathbf{10}}$ representation by the requirement of Bose symmetrization. This implies that the $\pi_1(1400)$ must belong to a family of four-quark states $(q\bar{q} + q\bar{q})$. In contrast, a second exotic meson, the $\pi_1(1600)$ with $I^G(J^{PC}) = 1^-(1^{-+})$, is reported to have a substantial decay mode into $\eta'\pi$ [3]. Whatever its constituent quark content may be—since the η' is mostly an $SU(3)$ singlet—it is a member of an octet.

The multiplet $\mathbf{10} \oplus \overline{\mathbf{10}}$ is given in a pictorial form in Fig.1.

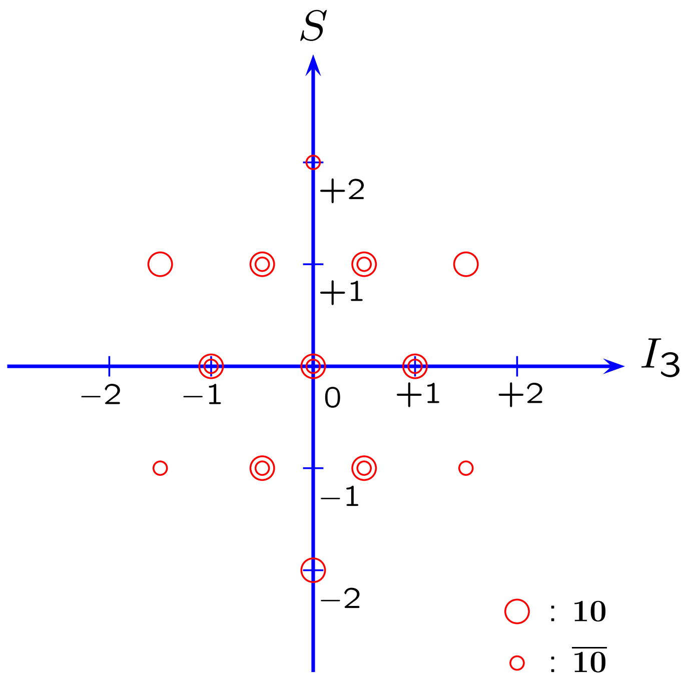

Figure 1: Weight Diagram for multiplet $\mathbf{10}\oplus\overline{\mathbf{10}}$; single circles have just one member of the multiplet, while the double circles indicate two occupancies by the members of the multiplet.

The purpose of this letter is explore the consequence of the existence of other vector mesons predicted to exist in the multiplet $\mathbf{10} \oplus \overline{\mathbf{10}}$ [2]. In the limit of flavor $SU(3)$ symmetry, *the mass of the* $I^G(J^{PC}) = 1^+(1^{--})$ *member must be equal to that of the* $\pi_1(1400)$. We shall adopt the notation $\rho_x(1400)$ for this state. We know that *both* must belong to the $q\bar{q}q\bar{q}$ family of mesons.

A full account of the $J^{PC} = 1^{--}$ and 1^{-+} 'vector' mesons with $q\bar{q}q\bar{q}$ entails existence of *two* sets of 'supermultiplets' with 81 members[a] in each. Each supermultiplet has the structure

$$\mathbb{V} = \{q\bar{q}q\bar{q}\} = 2\times\mathbf{1}\ \oplus\ 4\times\mathbf{8}\ \oplus\ \mathbf{10}\ \oplus\ \overline{\mathbf{10}}\ \oplus\ \mathbf{27} \tag{1}$$

where we consider here three flavors $q = \{u, d, s\}$. Of course, not all members of the multiplets need to support quasi-bound-state systems—with reasonably finite widths—which can be identified experimentally.

Let χ^0 be the wave function for a nonstrange, neutral state in any of the representations listed above (there is at least one such state in each). We now define the supermultiplets through the charge-conjugation operator $\mathbb{C}$

$$\begin{aligned}\mathbb{V}_\zeta:\quad & \mathbb{C}\,|\chi^0(\boldsymbol{n})\rangle = \zeta\,|\chi^0(\boldsymbol{n})\rangle, \qquad \boldsymbol{n} = \mathbf{1},\ \mathbf{8},\ \mathbf{27}\\ & \mathbb{C}\,|\chi^0(\mathbf{10})\rangle = \zeta\,|\chi^0(\overline{\mathbf{10}})\rangle, \quad \mathbb{C}\,|\chi^0(\overline{\mathbf{10}})\rangle = \zeta\,|\chi^0(\mathbf{10})\rangle\end{aligned} \tag{2}$$

where $\zeta = \pm 1$. Thus, ζ is simply the $\mathbb{C}$ eigenvalue of a χ^0 which belongs to any of the 'self-conjugate' representations $\mathbf{1}$, $\mathbf{8}$ and $\mathbf{27}$. The subscript therefore refers to the C-parity of the dominant J^{PC} in the family ($\zeta = C$). Each $\mathbb{V}_\pm$ supermultiplet comes with 61 $J^{PC} = 1^{-\pm}$ members belonging to the self-conjugate representations of $SU(3)$. The $\mathbf{10}\oplus\overline{\mathbf{10}}$ members are broken up into 14 $J^P = 1^-$ strange members and *two* sets of three nonstrange $I^G(J^{PC}) = 1^\pm(1^{-\mp})$ members. Thus the physical states in eigenstates of $\mathbb{C}$ are

$$\chi^0_c = \frac{1}{\sqrt{2}}\left[\chi^0(\mathbf{10}) + c\,\zeta\,\chi^0(\overline{\mathbf{10}})\right], \quad \mathbb{C}|\chi^0_c\rangle = c\,|\chi^0_c\rangle \tag{3}$$

where $c = \pm 1$ is the the C-parity eigenvalue of X. The 'quantum number' ζ has *nothing* to do with the C-parity of the nonstrange members of $\mathbf{10}\oplus\overline{\mathbf{10}}$. We assume that one supermultiplet, $\mathbb{V}_+$ or $\mathbb{V}_-$, gives rise to $\pi_1(1400)$ and $\rho_x(1400)$; we shall designate their counterparts by $\pi_1'(1400?)$ and $\rho_x'(1400?)$ in the other supermultiplet. All four states are isospin triplets and belong to $\mathbf{10}\oplus\overline{\mathbf{10}}$ representations.

Let X stand for a nonstrange state belonging to a $\mathbf{10}\oplus\overline{\mathbf{10}}$ representation. The main purpose of this paper is to point out that the amplitude for the decay $X \to \{a\} + \{b\}$ where $\{a\}$ and $\{b\}$ are members of the octets (not necessarily the same) is *independent* of c. For the purpose, consider the decay modes of a $J^P = 1^-$ nonstrange meson X as outlined below:

$$X \to [\{\pi\}\{\pi\}]_P\,,\ [\{\rho\}\{\rho\}]_{P,F}\,,\ [\{a_1\}\{\pi\}]_{S,D}\,,\ [\{a_2\}\{\pi\}]_D\,, \tag{4a}$$

[a] The number of states in a supermultiplet can derived in two different ways: $\{q\bar{q}+q\bar{q}\} = (\mathbf{1}\oplus\mathbf{8})\otimes(\mathbf{1}\oplus\mathbf{8})$ or $\{qq+\bar{q}\bar{q}\} = (\overline{\mathbf{3}}\oplus\mathbf{6})\otimes(\mathbf{3}\oplus\overline{\mathbf{6}})$.

$$\rightarrow [\{\rho\}\{\pi\}]_P\,, [\{b_1\}\{\pi\}]_{S,\,D} \tag{4b}$$

where the bracket { } refers to the totality of an octet. For example, $\{\pi\}$ stands for π, K, $\bar{K}$ and $[\eta]_8$. (we use $[\ ]_8$ to denote the octet component of a particle; in addition, we follow a common practice of writing $[a\,b]_L$ to indicate an orbital angular momentum L between a and b.) The $SU(3)$ wave functions ϕ (real by definition) for nonstrange neutral members of $\{a\}\,\{b\}[4]$ are transformed,[b] under $\mathbb{C}$,

$$\mathbb{C}\,\phi(\overline{\mathbf{10}}) = g\,\phi(\mathbf{10}), \quad \mathbb{C}\,\phi(\mathbf{10}) = g\,\phi(\overline{\mathbf{10}}) \tag{5}$$

where $g = C_a \cdot C_b = \pm 1$. C refers to the C-parity of the octets involved in the final state. Hence g is equal to a product of two G-parities for the *isovector* members of $\{a\}$ and $\{b\}$. For example, $g = +1$ for the decay $\{\pi\}\{\pi\}$ and likewise for all the final states listed in (4a), while $g = -1$ for (4b). Define

$$\phi_c = \frac{1}{\sqrt{2}}\left[\phi(\mathbf{10}) + c\,g\,\phi(\overline{\mathbf{10}})\right], \quad \mathbb{C}|\phi_c\rangle = c\,|\phi_c\rangle \tag{6}$$

So ϕ_c has the same eigenvalue c under $\mathbb{C}$ as does χ^0_c.

Because of C-parity conservation, the amplitudes for the decay $X \rightarrow a + b$ must obey the relationship[c]

$$\langle\phi(\mathbf{10})|\mathcal{M}|\chi^0(\mathbf{10})\rangle = \zeta\,g\,\langle\phi(\overline{\mathbf{10}})|\mathcal{M}|\chi^0(\overline{\mathbf{10}})\rangle \tag{7}$$

The C-parity conservation requires that $\mathbb{C}$ commutes with $\mathcal{M}$. We now insert $\mathbb{C}^\dagger\,\mathbb{C} = I$ next to the operator $\mathcal{M}$ on the right side of (7) and use (2) and (5)—to see that (7) is indeed an identity. Now examine the decay amplitude for $X \rightarrow \{a\} + \{b\}$ in eigenstates of $\mathbb{C}$

$$A_c = \langle\phi_c|\mathcal{M}|\chi^0_c\rangle \tag{8}$$

Applying (3), (6) and (7), we find

$$A_c = \langle\phi(\mathbf{10})|\mathcal{M}|\chi^0(\mathbf{10})\rangle = \zeta\,g\,\langle\phi(\overline{\mathbf{10}})|\mathcal{M}|\chi^0(\overline{\mathbf{10}})\rangle \tag{9}$$

so that the decay amplitude is indendent of c. Or, equivalently, $A_+ = A_-$. *This is our main result.*

[b] There is an overall phase η in the defining formula (8.2) of Ref.[4], which has been set to +1.

[c] In our original article, we have introduced an arbitrary constant $\kappa = \pm 1$ instead of the factor $\zeta\,g$ in the formula. This is an assumption which we now believe is not justifiable (see acknowledement).

The coupling constants f for the two C-parity eigenstates are summarized in Tables I and II. If we neglect mass-dependent factors, such as the angular-momentum barrier factors and the phase-space elements, then the squares of the coupling constants should correspond to the branching ratios.

Table I: Coupling Constants f for $X \to \{\pi\}\{\pi\}$ [a]

Name	$I^G(J^{PC})$	$\sqrt{2}\,\chi_c$	$\sqrt{2}\,\phi_c$	$f(\pi\,\pi)$	$f(K\bar{K})$	$f(\pi[\eta]_8)$
$\pi_1(1400)$	$1^-(1^{-+})$	$\chi(\mathbf{10}) + \zeta\,\chi(\overline{\mathbf{10}})$	$\phi(\mathbf{10}) + \phi(\overline{\mathbf{10}})$	0	0	1
$\rho_x(1400)$	$1^+(1^{--})$	$\chi(\mathbf{10}) - \zeta\,\chi(\overline{\mathbf{10}})$	$\phi(\mathbf{10}) - \phi(\overline{\mathbf{10}})$	$\sqrt{1/3}$	$\sqrt{2/3}$	0

[a] The same branching ratios apply to the decay modes $\{\rho\}\{\rho\}$, $\{a_1\}\{\pi\}$ and $\{a_2\}\{\pi\}$.

Table II: Coupling Constants f for $X \to \{\rho\}\{\pi\}$ [a]

Name	$I^G(J^{PC})$	$\sqrt{2}\,\chi_c$	$\sqrt{2}\,\phi_c$	$f(\rho\,\pi)$	$f(K^*\bar{K})$	$f(\pi[\omega]_8)$
$\pi_1(1400)$	$1^-(1^{-+})$	$\chi(\mathbf{10}) + \zeta\,\chi(\overline{\mathbf{10}})$	$\phi(\mathbf{10}) - \phi(\overline{\mathbf{10}})$	$\sqrt{1/3}$	$\sqrt{2/3}$	0
$\rho_x(1400)$	$1^+(1^{--})$	$\chi(\mathbf{10}) - \zeta\,\chi(\overline{\mathbf{10}})$	$\phi(\mathbf{10}) + \phi(\overline{\mathbf{10}})$	0	0	1

[a] The same branching ratios apply to the decay mode $\{b_1\}\{\pi\}$.

It should be noted that a state X cannot couple to both $\pi\pi$ and $\pi\eta$ because of the G-parity conservation. The same comments apply to $\rho\pi$ and $\pi\omega$. What is really new in Tables I and II may be restated as follows:

$$\begin{aligned} f\,\{\pi_1(1400) \to \pi[\eta]_8\} &= \sqrt{3}\,f\,\{\rho_x(1400) \to \pi\pi\} \\ f\,\{\rho_x(1400) \to \pi[\omega]_8\} &= \sqrt{3}\,f\,\{\pi_1(1400) \to \rho\pi\} \end{aligned} \tag{10}$$

Note also that these relations apply equally well to the $\pi_1'(1400?)$ and $\rho_x'(1400?)$, i.e. the results are independent of ζ.

In summary, we expect two states of $I^G(J^{PC}) = 1^+(1^{-+})$ mesons, $\pi_1(1400)$ and $\pi_1'(1400?)$, and their vector partners with $I^G(J^{PC}) = 1^-(1^{--})$, $\rho_x(1400)$ and $\rho_x'(1400?)$, all belonging to $\mathbf{10} \oplus \overline{\mathbf{10}}$ representations of flavor $SU(3)$. If the $\pi_1(1400)$ and $\rho_x(1400)$ belong in the supermultiplet $\mathbb{V}_\pm$, then the $\pi_1'(1400?)$ and $\rho_x'(1400?)$ are members of $\mathbb{V}_\mp$. Since the $\pi_1(1400)$

is seen to decay into $\eta\pi$, its partner $\rho_x(1400)$ should decay into $\pi\pi$ with about one-third the strength of $\pi_1(1400) \to \eta\pi$. The $\rho_x(1400)$ should also couple to $K\bar{K}$, with a relative branching ratio $B(\pi\pi)/B(K\bar{K}) = 1/2$. The Crystal Barrel Collaboration reported[5] observation of a $I^G(J^{PC}) = 1^+(1^{-+})$ state[d] decaying into $\rho\pi$. If this state is different from the $\pi_1(1400)$ in the supermultiplet $\mathbb{V}_\pm$, then it could be the $\pi_1'(1400?)$ in $\mathbb{V}_\mp$. In this case, its partner $\rho_x'(1400?)$ should couple to $\pi\omega$ with about three-times the strength of $\pi_1'(1400?) \to \rho\pi$.

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[d] The results are consistent with a single state with mass at ~ 1450 MeV and its width fixed at 310 MeV; however, there are in fact indications of two states at 1405 and 1650 MeV.

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The reader may consult the references therein for a comprehensive list of previous publications on this topic.

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