The Use of Control Regions to Subtract Background Events in PWA

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Abstract

Given is an overview of the extended maximum likelihood fit and a description on using control regions to subtract out the contribution of background events from the extended maximum likelihood function. The results of applying this technique to an $\eta\eta$ analysis is presented.

Introduction

In a PWA of a many-body final state, one sometimes takes advantage of narrow resonances to greatly simplify the analysis. Often associated with these narrow states are non-negligible backgrounds. The ω or η decaying to $\pi^+\pi^-\pi^\circ$ are examples of narrow states whose signal can have large 3π background. It is important to account for or eliminate this background in the PWA since such large backgrounds could simulate or blur out physical structure in some particular partial wave. For signals such as the ω, which has non-zero spin, the PWA can use the decay angular information to separate the signal from the background events. Alternatively, or in addition to, one can include in the PWA fit a sample of events from control regions (sidebands) to subtract out the contribution of background events. The intent of this note is to provide a description for such a background subtraction within the context of an extended maximum likelihood analysis.

Extended Maximum Likelihood Fit

For completeness, a review of the extended maximum likelihood fitting is given first in order to provide a foundation for later discussion.

The intensity distribution is characterized by a set of interfering (denoted by α) and non-interfering (denoted by ϵ and k) orthonormal amplitudes factored into a production part (denoted by V) and a decay part (denoted by A):

$$I(\tau) = \sum_{\epsilon,k} |\sum_{\alpha} {}^{\epsilon}V_{k\alpha}\, {}^{\epsilon}A_{\alpha}(\tau)|^2 \tag{1}$$

These amplitudes are expressed in the reflectivity basis which introduces the reflectivity ϵ. The subscript k represents the spin orientation in the production process, α is the partial wave index, and τ represents all of the variables necessary to have a complete description of the final state such as kinematic variables. Binning the data in total effectives mass and in t removes the τ dependence in the production amplitudes.

The calculation of the decay amplitudes is based primarily on well known laws of physics with limited phenomenological input from the Isobar Model. The production amplitudes are complex parameters which are varied to best fit the observed intensity distribution.

The likelihood function for finding N_o observed events in a given bin is:

$$\mathcal{L} = e^{-\overline{N_o}({}^{\epsilon}V_{k\alpha})} \prod_{i}^{N_o} F({}^{\epsilon}V_{k\alpha}; \tau_i), \tag{2}$$

where the probability of observing an event "i' is given by

$$F({}^{\epsilon}V_{k\alpha}; \tau_i) = I(\tau_i) = \sum_{k,\epsilon,\alpha,\alpha'} {}^{\epsilon}V_{k\alpha}\, {}^{\epsilon}V^*_{k\alpha'}\, {}^{\epsilon}A_{\alpha}(\tau_i)\, {}^{\epsilon}A^*_{\alpha'}(\tau_i) \tag{3}$$

and

$$\overline{N_o}({}^{\epsilon}V_{k\alpha}) = \int I(\tau)\eta(\tau)\, dLips = \sum_{k,\epsilon,\alpha,\alpha'} {}^{\epsilon}V_{k\alpha}\, {}^{\epsilon}V^*_{k\alpha'} \int {}^{\epsilon}A_{\alpha}(\tau)\, {}^{\epsilon}A^*_{\alpha'}(\tau)\eta(\tau)\, dLips \tag{4}$$

is the average number of events one would observe if the same experiment were repeated many times[1]. The experimental acceptance is represented by $\eta(\tau)$, and $dLips$ represent the Lorentz-invariant phase-space element.

The integrals over the decay amplitudes are often called the accepted normalization integrals[2],

$${}^{\epsilon}\Phi^{\eta}_{\alpha\alpha'} = \int {}^{\epsilon}A_{\alpha}(\tau)\, {}^{\epsilon}A^*_{\alpha'}(\tau)\eta(\tau)\, dLips. \tag{5}$$

Since the acceptance $\eta(\tau)$ is not known analytically, the accepted normalization integrals are evaluated using Monte Carlo events where the effects of the experiment are modeled to account for the experimental acceptance:

$$ {}^{\epsilon}\Phi^{\eta}_{\alpha\alpha'} = \frac{1}{N_r} \sum_{i}^{N_\eta} {}^{\epsilon}A_{\alpha}(\tau_i)\, {}^{\epsilon}A^{*}_{\alpha'}(\tau_i). \tag{6} $$

The sum is over the accepted Monte Carlo data, where N_r is the number of raw Monte Carlo events generated to end up with a set of N_η accepted events. The phase-space factor is taken into account by generating the Monte Carlo events according to phase-space.

The extended log-likelihood function which is maximized now becomes:

$$ ln\mathcal{L} = \sum_{i}^{N_o} ln(F({}^{\epsilon}V_{k\alpha};\tau_i)) - \overline{N_o}({}^{\epsilon}V_{k\alpha}) \tag{7} $$

$$ ln\mathcal{L} = \sum_{i}^{N_o} ln(\sum_{k,\epsilon,\alpha,\alpha'} {}^{\epsilon}V_{k\alpha}\, {}^{\epsilon}V^{*}_{k\alpha'}\, {}^{\epsilon}A_{\alpha}(\tau)\, {}^{\epsilon}A^{*}_{\alpha'}(\tau)) - \sum_{k,\epsilon,\alpha,\alpha'} {}^{\epsilon}V_{k\alpha}\, {}^{\epsilon}V^{*}_{k\alpha'}\, {}^{\epsilon}\Phi^{\eta}_{\alpha\alpha'}. \tag{8} $$

In practice, the production amplitudes are renormalized by setting

$$ V' = \frac{V}{\sqrt{N_o}} \text{ then renaming } V' \to V \tag{9} $$

so that now

$$ \sum_{k,\epsilon,\alpha,\alpha'} {}^{\epsilon}V_{k\alpha}\, {}^{\epsilon}V^{*}_{k\alpha'}\, {}^{\epsilon}\Phi^{\eta}_{\alpha\alpha'} = \frac{\overline{N_o}}{N_o}. \tag{10} $$

Neglecting the constant log term, the log-likelihood function reads:

$$ ln\mathcal{L} = \sum_{i}^{N_o} ln(\sum_{k,\epsilon,\alpha,\alpha'} {}^{\epsilon}V_{k\alpha}\, {}^{\epsilon}V^{*}_{k\alpha'}\, {}^{\epsilon}A_{\alpha}(\tau)\, {}^{\epsilon}A^{*}_{\alpha'}(\tau)) - N_o \sum_{k,\epsilon,\alpha,\alpha'} {}^{\epsilon}V_{k\alpha}\, {}^{\epsilon}V^{*}_{k\alpha'}\, {}^{\epsilon}\Phi^{\eta}_{\alpha\alpha'}. \tag{11} $$

It is advantageous to re-normalize because, to first order, the renormalized production amplitudes are independent of variations in size among different bins, and therefore they are excellent starting values for neighboring bins[2].

Background Subtraction

The non-signal background can be simply subtracted from the likelihood by using events from control bands that are defined from adjacent regions in the signal spectra. These control band events are assigned a negative weight, w_j, with a requirement that the sum of the weights equal in magnitude to the number of estimated background events under the signal (i.e. signal events have weight equal 1).

Clearly, the total intensity is the sum of the two: $I_{total} = I_{signal} + I_{background}$. Then the likelihood function for finding N_s observed signal events in a given bin is:

$$\mathcal{L} = e^{-\overline{N_s}({}^{\epsilon}V_{k\alpha})} \frac{\prod_i^{N_o} I(\tau_i)}{\prod_j^{N_b} |w_j| \, I(\tau_j)} \tag{12}$$

The new factor in the denominator is the product of probabilities for the control band events. At this point one can see that if the control band events are a reasonable approximation of the background events then on average the ratio of background event probabilities to control band event probabilities is 1 –thus canceling the contribution of the background events to the likelihood. The extended log-likelihood function now becomes:

$$\begin{aligned} ln\mathcal{L} = \sum_i^{N_o} ln(\sum_{k,\epsilon,\alpha,\alpha'} {}^{\epsilon}V_{k\alpha} \, {}^{\epsilon}V^*_{k\alpha'} \, {}^{\epsilon}A_{\alpha}(\tau_i) \, {}^{\epsilon}A^*_{\alpha'}(\tau_i)) \\ + \sum_j^{N_b} w_j \, ln(\sum_{k,\epsilon,\alpha,\alpha'} {}^{\epsilon}V_{k\alpha} \, {}^{\epsilon}V^*_{k\alpha'} \, {}^{\epsilon}A_{\alpha}(\tau_j) \, {}^{\epsilon}A^*_{\alpha'}(\tau_j)) \\ - \sum_{k,\epsilon,\alpha,\alpha'} {}^{\epsilon}V_{k\alpha} \, {}^{\epsilon}V^*_{k\alpha'} \, {}^{\epsilon}\Phi^{\eta}_{\alpha\alpha'}. \end{aligned} \tag{13}$$

where w_j is the control band event weight.

For a simple control band (i. e. non-crossing bands) where the number of control band events equals the number of estimated background events (w_j equal to -1), the renormalized log-likelihood is:

$$\begin{aligned} ln\mathcal{L} = \sum_i^{N_o} ln(\sum_{k,\epsilon,\alpha,\alpha'} {}^{\epsilon}V_{k\alpha} \, {}^{\epsilon}V^*_{k\alpha'} \, {}^{\epsilon}A_{\alpha}(\tau_i) \, {}^{\epsilon}A^*_{\alpha'}(\tau_i)) \\ - \sum_j^{N_b} ln(\sum_{k,\epsilon,\alpha,\alpha'} {}^{\epsilon}V_{k\alpha} \, {}^{\epsilon}V^*_{k\alpha'} \, {}^{\epsilon}A_{\alpha}(\tau_j) \, {}^{\epsilon}A^*_{\alpha'}(\tau_j)) \\ -N_s \sum_{k,\epsilon,\alpha,\alpha'} {}^{\epsilon}V_{k\alpha} \, {}^{\epsilon}V^*_{k\alpha'} \, {}^{\epsilon}\Phi^{\eta}_{\alpha\alpha'}. \end{aligned} \tag{14}$$

where $N_s = N_o - N_b$.

As a final remark, when comparing the PWA results using predicted data, one must also perform a background subtraction of the data spectra.

$\eta\eta$ PWA

A fit of the $\eta\eta$ system was performed for the 1995 data $\pi^- p \to n\pi^+\pi^-\pi^\circ\eta$. The largest contribution to the background is due the non-η events in the $\eta \to \pi^+\pi^-\pi^\circ$ signal. Figure 1 shows the $\pi^+\pi^-\pi^\circ$ mass spectrum for the η and control bands selections. The non-η signal was estimated to be about 15% of

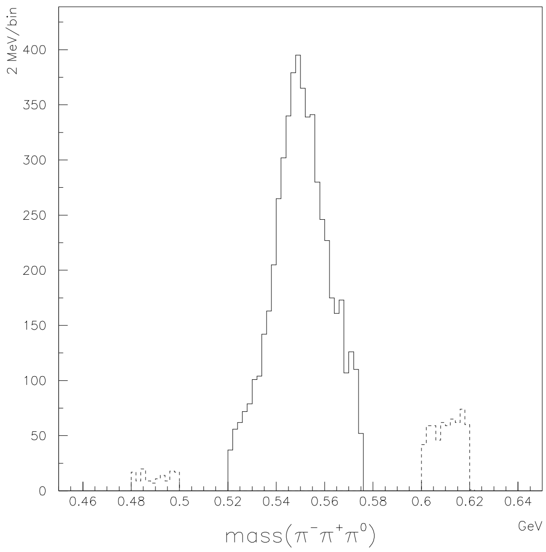

Figure 1: The $\pi^+\pi^-\pi^\circ$ mass spectrum for the η and control bands selections.

the total signal within the η selection of 520 MeV to 575 MeV. Control bands were selected with a total number of events equal to the estimated number of non-η events under the η signal. The $\eta\eta$ mass spectrum is show in Figure 2 along with the estimated background from the control bands. The PWA was performed in 60 MeV mass bins from 1060 MeV to 2140 MeV with t between 0 and 1.2 GeV^2. The partial waves included in the fit were:

- $0^{++}0^{-}S$
- $2^{++}0^{-}D$
- $2^{++}0^{-}D$
- $2^{++}1^{-}D$
- $2^{++}1^{+}D$

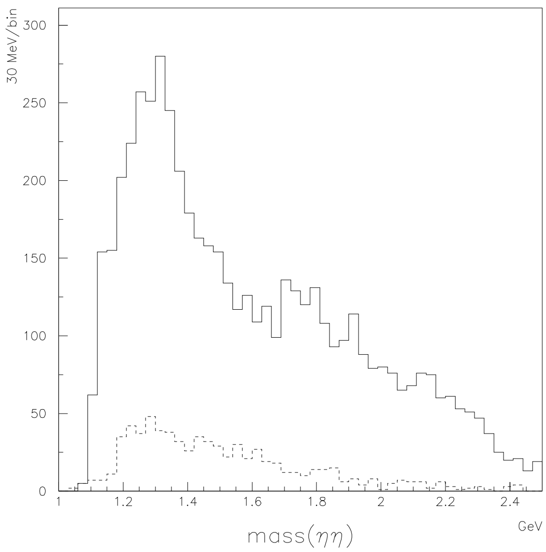

Figure 2: The $\eta\eta$ mass spectrum for the η and control bands (dashed) selections.

The fit results are shown in Figures 3 and 4. The black points represent fits without background subtraction whereas the green points represent fits with background subtraction. Displayed are actually the results of 40 fits: 20 fits with and 20 fits without background subtraction. Random starting values were used for each fit. The $\eta\eta$ system has ambiguous solutions. The solutions fall into two types and differ mostly in the 0^{++} intensity. The background subtracted fits as expected exhibit lower intensities consistent with our expectations. To first order the different fit results agree, which indicates that the background events do not have a large affect on the fit. This is also what we expected since our results agree with $\eta\eta$ results analyzed via a different channel which does not suffer from the same type of background. Since our analysis is in progress, we leave the physical interpretation of the results for a later discussion.

References

[1] Orear, Jay, *Notes on Statistics For Physicists, Revised* **CLNS 82/511** (1982).

[2] Chung, S. U., *Brookhaven Report* **BNL-QGS-93-05** (1995).

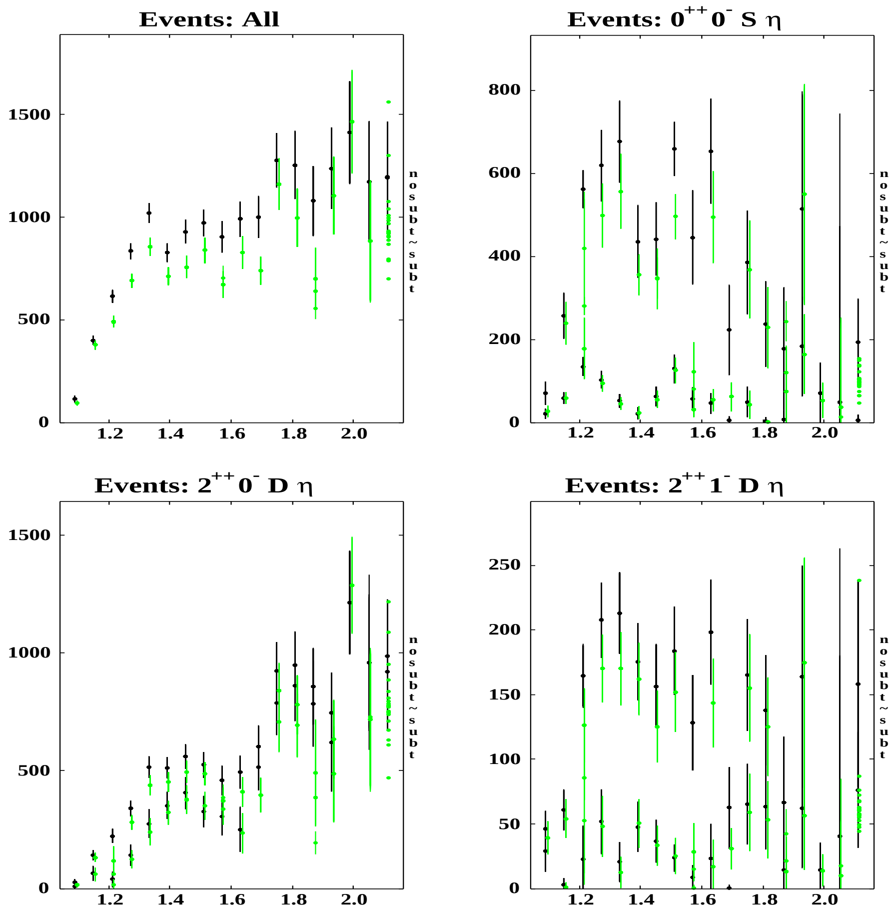

Figure 3: The PWA fit results for $\eta\eta$. The black points represent fits without background subtraction whereas the green points represent fits with background subtraction. Displayed are actually the results of 40 fits: 20 fits with and 20 fits without background subtraction. Random starting values were used for each fit.

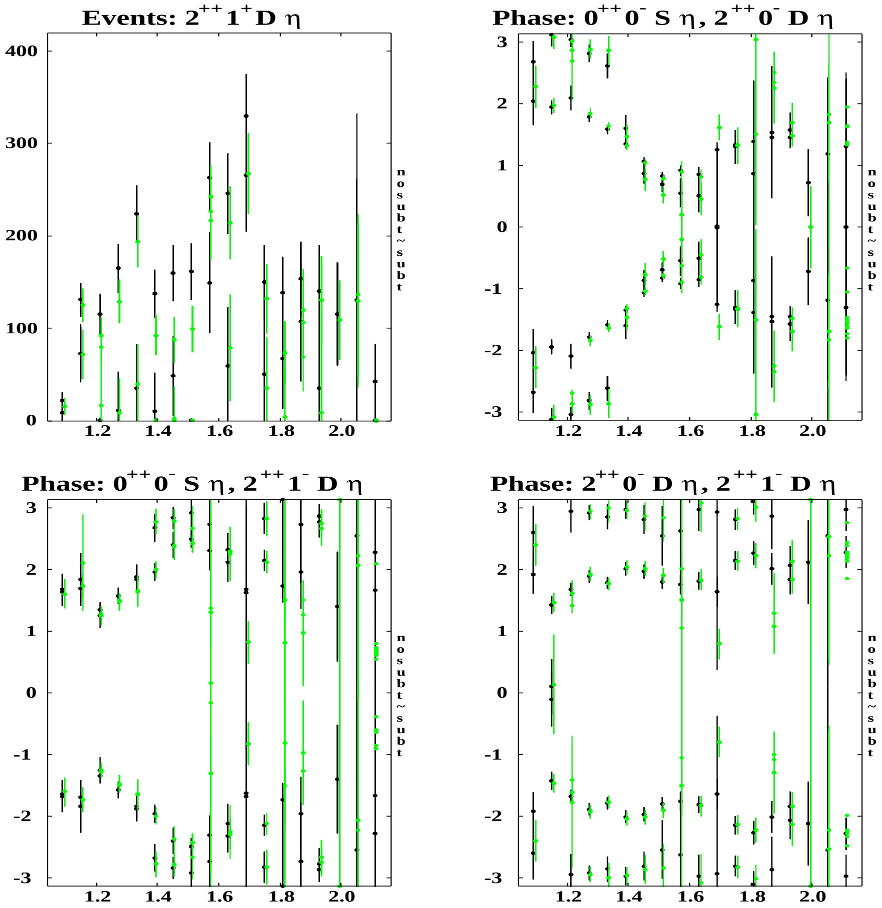

Figure 4: The PWA fit results for $\eta\eta$. The black points represent fits without background subtraction whereas the green points represent fits with background subtraction. Displayed are actually the results of 40 fits: 20 fits with and 20 fits without background subtraction. Random starting values were used for each fit.