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Gary Adams
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Dear Gary,

It occurs to me that I should clarify my formula for the mass-dependent width.

Relevant references are given below:

for *one* "derivation," see S. U. Chung,
"Formulas for Partial-Wave Analysis—Version II,"
`http://www.phy.bnl.gov/~e852/reviews.html`
S. U. Chung, *et al.*, Ann. Physik **4**, 404 (1995)

Let me take an example of $\rho \to \pi\pi$. The width is given by

$$\Gamma(m) = \Gamma_0 \left(\frac{m_0}{m}\right) \left(\frac{p}{p_0}\right) \left[\frac{F_1(p)}{F_1(p_0)}\right]^2 \tag{1}$$

where $F_1(p)$ is the Blatt-Weisskopf barrier factor for a p-wave, $m_0 = 769$ MeV and $p_0 = 358$ MeV. This formula can be cast into

$$\Gamma(m) = \Gamma_0 \left(\frac{m_0}{m}\right) \left(\frac{p}{p_0}\right)^3 \left(\frac{p_0^2 + p_R^2}{p^2 + p_R^2}\right) \tag{2}$$

where $p_R = 197$ MeV.

The mass dependence in the above formula comes from an interplay between unitarity and Lorentz invariance. More specifically, it comes about because one needs to employ the simplest 'analytic' invariant transition amplitude for the process $\pi\pi \to \rho \to \pi\pi$. So the formula is 'correct' for a P-wave $\pi\pi$ elastic scattering at the mass of the ρ. It is also the 'right formula' for an S-wave (or a D-wave) $\pi\omega$ elastic scattering for the $J^P = 1^+$ partial wave at the $b_1(1235)$ mass. (Note that its S and D waves are close to being relatively real.)

As $m \to \infty$, the width formula becomes

$$\Gamma(m) \to \Gamma_0 \left(\frac{m_0}{p_0}\right) \left(\frac{p_0^2 + p_R^2}{2p_0^2}\right) = \text{const} \tag{3}$$

The mass dependence of the width formula specifies how it should behave near the threshold (i.e. $p \simeq 0$); we do not know how it should behave as $m \to \infty$. It seems reasonable to me that, in a phenomenological approach, one should simply put it to a constant.

We should *not* be changing our width formula in the midst of a series of publications from E852. If you insist that the Jackson prescription is the right one, then I propose that we try both formulas and adjust the systematic errors accordingly. And we should clearly point out what we have done in all subsequent publications.

Incidentally, we should always remember that the width which appear in the denominator of a Breit-Wigner form refers to the *total* width. If a high-mass resonance decays into many open channels, then the total width is a sum of all the allowed channels. We do not in general know all of the open channels nor their decay strengths. Given this situation, a sensible phenomenological approach might be take the total width as a constant in the Breit-Wigner form. If we decide to take this route, then it should also be clearly stated in our publications.

Sincerely yours,

Suh-Urk Chung