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Determining the Gluonic Content of Isoscalar Mesons

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Abstract

We develop tools to determine the gluonic content of a resonance of known mass, width and J^{PC} from its branching fraction in radiative quarkonium decays and production cross section in $\gamma\gamma$ collisions. We test the procedures by applying them to known $q\bar{q}$ mesons, then analyze four leading glueball candidates. We identify inconsistencies in data for $J/\psi \to \gamma f_0(1500)$ and $J/\psi \to \gamma f_J(1710)$ whose resolution can quantify their glueball status.When $\Gamma(f_0(1500) \to \gamma\gamma)$ and $\Gamma(f_J(1710) \to \gamma\gamma)$ are known, the $n\bar{n}, s\bar{s}, gg$ mixing angles can be determined. The enigmatic situation in 1400-1500 MeV region of the isosinglet 0^{-+} sector is discussed.

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1 Introduction

There has been considerable recent interest in the possible sighting of glueballs. Four states are of particular interest:

- $f_0(1500)$[1, 2, 3]
- $f_J(1710)$[4] where $J = 0$ or 2[5]
- $\xi(2230)$[6]
- $\eta(1440)$[7], now resolved into two pseudoscalars.

In this paper we calculate the production rate of conventional mesons ($q\bar{q}$) and glueballs ("G") in the radiative decay of vector quarkonium, as a function of their mass, angular momentum, and width. If the data on the radiative production of these states are correct, we find that

(i) The $f_0(1500)$ is probably produced at a rate too high to be a $q\bar{q}$ state. The average of world data suggests it is a glueball-$q\bar{q}$ mixture.

(ii) The $f_J(1710)$ is produced at a rate which is consistent with it being $q\bar{q}$, only if $J = 2$. If $J = 0$, its production rate is too high for it to be a pure $q\bar{q}$ state but is consistent with it being a glueball or mixed $q\bar{q}$-glueball having a large glueball component.

(iii) The $\xi(2230)$, whose width is $\sim$ 20 MeV, is produced at a rate too high to be a $q\bar{q}$ state for either $J = 0$ or 2. If $J = 2$, it is consistent with being a glueball. The assignment $J = 0$ would require $Br(J/\psi \to \gamma\xi) \lesssim 3\ 10^{-4}$, which already may be excluded.

(iv) The enhancement once called $\eta(1440)$ has been resolved into two states. The higher mass $\eta(1480)$ is dominantly $s\bar{s}$ with some glue admixture, while the lower state $\eta(1410)$ has strong affinity for glue.

We note what improvements in data would allow these constraints to be sharpened. We also analyze 3-state mixing in the 0^{++} sector between $n\bar{n}$, $s\bar{s}$ and gg, showing how $\Gamma(J/\psi \to \gamma f_0)$ and $\Gamma(f_0 \to \gamma\gamma)$ determine mixing angles.

The interest in these states as glueball candidates is motivated on both phenomenological and theoretical grounds. Phenomenologically, these states satisfy qualitative criteria expected for glueballs[8]:

1. Glueballs should be favoured over ordinary mesons in the central region of high energy scattering processes, away from beam and target quarks. The $f_J(1710)$ and possibly the $f_0(1500)$ have been seen in the central region in pp collisions[9, 10].
2. Glueballs should be produced in proton-antiproton annihilation, where the destruction of quarks creates opportunity for gluons to be manifested. This is the Crystal Barrel [11], and E760 [12] production mechanism, in which detailed decay systematics of $f_0(1500)$ have been studied. The empirical situation with regard to $f_J(1710)$ and $\xi(2230)$ is currently under investigation. The $\eta(1440)$ is clearly seen in $p\bar{p}$ annihilation[13, 14]
3. Glueballs should be enhanced compared to ordinary mesons in radiative quarkonium decay. In fact, all four of these resonances are produced in radiative J/ψ decay at a level typically of ~ 1 part per thousand. A major purpose of this paper is to decide whether these rates indicate that these resonances are glueballs, or not.

On the theoretical side, lattice QCD predicts that the lightest "ideal" (i.e., quenched approximation) glueball be 0^{++}, with state-of-the-art mass predictions of 1.55 ± 0.05 GeV[15] and 1.74 ± 0.07 GeV[4]. That lattice QCD is now concerned with such fine details represents considerable advance in the field and raises both opportunity and enigmas. First, it encourages serious consideration of the further lattice predictions that the 2^{++} glueball lie in the 2.2 GeV region, and hence raises interest in the $\xi(2230)$. Secondly, it suggests that scalar mesons in the $1.5 - 1.7$ GeV region merit special attention. Amsler and Close[2] have pointed out that the $f_0(1500)$ shares features expected for a glueball that is mixed with the nearby isoscalar members of the ${}^3P_0$ $q\bar{q}$ nonet. If the $f_J(1710)$ proves to have $J = 2$, then it is not a candidate for the ground state glueball and the $f_0(1500)$ will be essentially unchallenged. On the other hand, if the $f_J(1710)$ has $J = 0$ it becomes a potentially interesting glueball candidate. Indeed, Sexton, Vaccarino and Weingarten[16] argue that $f_{J=0}(1710)$ should be identified with the ground state glueball, based on its similarity in mass and decay properties to the state seen in their lattice simulation. While the consistency between theoretical mass predictions and the observed states is quite satisfactory in the 0^{++} and 2^{++} sectors, this is not the case in the 0^{-+} sector. Both lattice and sum rule calculations place the lightest 0^{-+} glueball at or above the 2^{++} glueball so that the appearance of a glueball-like pseudoscalar in the 1.4-1.5 GeV region is unexpected. It is interesting that its properties are consistent with those predicted for the gluino-gluino bound state in supersymmetry breaking scenarios with a light gluino[17]

In order to make quantitative estimates of the gluonic content of isosinglet mesons, we use their measured radiative quarkonium production rates and gamma-gamma decay widths. We apply the relationship proposed by Cakir and Farrar[18] (CF) between the branching fraction for a resonance R in radiative quarkonium decay, $b_{rad}(Q\bar{Q}_V \to \gamma + R) \equiv \Gamma(Q\bar{Q}_V \to \gamma + X)$ and its branching fraction to gluons, $br(R \to gg) \equiv \Gamma(R \to gg)/\Gamma(R \to all)$:

$$b_{rad}(Q\bar{Q}_V \to \gamma + R_J) = \frac{c_R x |H_J(x)|^2}{8\pi(\pi^2 - 9)} \frac{m_R}{M_V^2} \Gamma_{tot} br(R_J \to gg), \tag{1}$$

where M_V and m_R are masses of the initial and final resonances, and $x \equiv 1 - \frac{m_R^2}{M_V^2}$; c_R is a numerical factor and $H_J(x)$ a loop integral which will be discussed in section 2. For a resonance of known mass, total width (Γ_{tot}), and J^{PC}, a relationship such as eq. (1) would determine $br(R \to gg)$ if $b_{rad}(Q\bar{Q}_V \to \gamma + R)$ were known. CF argued that one expects

$$\begin{array}{rcl} br(R[q\bar{q}] \to gg) & = & 0(\alpha_s^2) \simeq 0.1 - 0.2 \\ br(R[G] \to gg) & \simeq & O(1). \end{array} \tag{2}$$

Thus knowledge of $br(R \to gg)$ would give quantitative information on the glueball content of a particular resonance. Using $H_J(x)$ determined in the non-relativistic quark model (NRQM), CF found that known $q\bar{q}$ resonances (such as $f_2(1270)$) satisfy the former and noted that the $f_0(1710)$ might be an example of the latter.

In the present paper we give a more general discussion of the functions $H_J(x)$ needed to employ Eq. (1), clarify some of the assumptions implicit in its derivation, and verify that application of the relation does not depend on flavor mixing. A number of experimental tests are proposed. We discuss the additional information that can be obtained when the cross section for production of the resonance in $\gamma\gamma$ collisions is known.

The paper is organized as follows. We start with a section on the formalism and its model dependence (sec. 2). A general treatment of the problem requires defining form factors for the coupling of a resonance, of specified J^{PC}, to a pair of virtual gluons. The partial width $\Gamma(R \to gg)$ fixes a linear combination of the form factors at the on-shell-gluon point. The internal structure of the resonance determines both the relative size of the various form factors at the on-shell-gluon point, as well as their virtuality dependence, just as in the case of the nucleon electromagnetic form factors F_1 and F_2. The $H_J(x)$'s depend on integrals of the form factors over the gluon virtualities. In sec. 2.4 we discuss higher order corrections, scale dependence, and the relationship of the $R \to gg$ form factors to the $R \to \gamma\gamma$ amplitudes which can in principle be measured in a photon-photon collider. (The phenomenology of the latter is developed in section 5). In sec. 3 we re-express eq. (1) so that its implications are more transparent and it is easier to apply to data. Our central results (eqs. 38-40) show how the spin of a resonance and its width into gluons fixes its production rate in radiative quarkonium decay. In section 3.2 we show that the relations do not depend on flavor mixing and that the known $q\bar{q}$ resonances $f_2(1270;\ 1525)$ satisfy eq.(2). In sec. 3.3 we discuss the utility of experimental study of radiative upsilon decay, especially $\Upsilon \to \gamma\chi$; in sec. 3.4 we investigate 1^{++} mesons. In secs. 4.1-4.3 we apply eqs. (38-40) to the $f_0(1500)$, $f_J(1710)$, $\xi(2300)$, and $\eta(1410; 1480)$ leading to the results listed at the beginning of the introduction. In section 5 we discuss how $\gamma\gamma \to R$ in combination with $J/\psi \to \gamma R$ can help to distinguish glueballs from $q\bar{q}$ states and determine basic parameters. Section 6 considers the possibility of glueball-$q\bar{q}$ mixing involving three states, $f_0(1370), f_0(1500)$ and $f_{0?}(1710)$. In general we use the nomenclature of the 1994 edition of the Particle Data Tables throughout[5]. Readers mainly interested in the phenomenological results can proceed directly to sections 3 et seq.

2 Formalism

2.1 $b_{rad}(Q\bar{Q}_V \to \gamma + R)$

The decay width for the radiative decay of a vector heavy quarkonium state, $Q\bar{Q}_V \to \gamma R$ is

$$\Gamma = \frac{1}{24\pi}\frac{k}{M_V^2}\sum_{i,f}|A|^2, \tag{3}$$

where k is the photon momentum, M_V is the mass of the spin-1 $Q\bar{Q}_V$ state, and the summation is over the polarizations of the initial and final particles. If the resonance R does not contain a "valence" $Q\bar{Q}$ component, the decay occurs through a two gluon intermediate state, in leading order pQCD, and the amplitude A is given by

$$A = \frac{1}{2}\sum\int\frac{d^4k}{(2\pi)^4}\frac{1}{k_1^2}\frac{1}{k_2^2} < (Q\bar{Q})_V|\gamma g^a g^b >< g^a g^b|R > . \tag{4}$$

The summation is over the polarization vectors $\epsilon_{1,2}$ and color indices a, b of the intermediate gluons, whose momenta are denoted $k_{1,2}$. The amplitude $< Q\bar{Q}_V|\gamma gg >$ couples a vector $Q\bar{Q}$ state with polarization and momentum (E, K) to a photon (ϵ, k) and the two virtual gluons. For heavy quarks, Q, this amplitude is reliably given by perturbative QCD. Using the non-relativistic quark model to describe the $Q\bar{Q}_V$ wavefunction (the pQCD-NRQM approximation[19, 20, 21]):

$$< Q\bar{Q}_V|\gamma g^a g^b >= e_Q g_s^2\delta^{ab}\sqrt{\frac{2}{3}}\frac{iR_V(0)}{\sqrt{4\pi M_V^3}}\frac{M_V^2}{k\cdot(k_1+k_2)k_1\cdot(k+k_2)k_2\cdot(k+k_1)}a_V, \tag{5}$$

where

$$a_V = \epsilon_1 \cdot \epsilon_2[-k_1 \cdot k\epsilon \cdot k_2 E \cdot k_1 - k_2 \cdot k\epsilon \cdot k_1 E \cdot k_2 - k_1 \cdot k k_2 \cdot k E \cdot \epsilon]$$
$$+E \cdot \epsilon[k_1 \cdot k\epsilon_1 \cdot k_2\epsilon_2 \cdot k + k_2 \cdot k\epsilon_2 \cdot k_1\epsilon_1 \cdot k - k_1 \cdot k_2\epsilon_1 \cdot k\epsilon_2 \cdot k]$$
$$+\{\epsilon_1, k_1 \Leftrightarrow \epsilon, k\} + \{\epsilon_2, k_2 \Leftrightarrow \epsilon, k\}. \tag{6}$$

$R_V(0)$ is the $(Q\bar{Q})_V$ wavefunction at the origin and e_Q is the charge of the heavy quark Q.

The amplitude $< g^a(k_1,\epsilon_1)g^b(k_2,\epsilon_2)|R >$ must be linear in ϵ_1 and ϵ_2 and Lorentz and gauge invariant. A linearly independent set of tensor structures satisfying these requirements for J^{++} states is given in [22]. Thus we can write, with the shorthand $G_{\mu\nu} \leftrightarrow k_\mu\epsilon_\nu - \epsilon_\mu k_\nu$ and the convention that $G^{1(2)}$ refers to $k_{1(2)}, \epsilon_{1(2)}$, and suppressing J^{PC} labels on the F_i's:

$$< g^a g^b|0^{++} > \;=\; \delta^{ab}\frac{A_{0^{++}}P_{\rho\sigma}}{\sqrt{3}}\left[F_1(k_1^2,k_2^2)G^1_{\mu\rho}G^2_{\nu\sigma} + F_2(k_1^2,k_2^2)k_1^\mu G^1_{\mu\rho}G^2_{\nu\sigma}k_2^\nu\right] \tag{7}$$

$$< g^a g^b|0^{-+} > \;=\; \delta^{ab}A_{0^{-+}}F_1(k_1^2,k_2^2)\epsilon^{\mu\nu\rho\sigma}G^1_{\mu\nu}G^2_{\rho\sigma}, \tag{8}$$

$$\begin{aligned} < g^a g^b|1^{++} > \;=\; & \delta^{ab}A_{1^{++}}\Big(F_1(k_1^2,k_2^2)\epsilon^{\mu\nu\rho\sigma}\epsilon_\sigma(G^1_{\mu\nu}G^2_{\rho\lambda}k_2^\lambda + G^2_{\mu\nu}G^1_{\rho\lambda}k_1^\lambda) \\ & +F_2(k_1^2,k_2^2)\epsilon^{\mu\nu\rho\sigma}\epsilon_\alpha(k_1^\alpha - k_2^\alpha)G^1_{\mu\nu}G^2_{\rho\sigma}\Big), \end{aligned} \tag{9}$$

$$\begin{aligned} < g^a g^b|2^{++} > \;=\; & \delta^{ab}A_{2^{++}}\epsilon_{\rho\sigma}\Big[F_1(k_1^2,k_2^2)G^1_{\mu\rho}G^2_{\mu\sigma} + F_2(k_1^2,k_2^2)k_1^\rho k_2^\sigma G^1_{\mu\nu}G^2_{\mu\nu} + \\ & F_3(k_1^2,k_2^2)k_1^\mu G^1_{\mu\rho}G^2_{\nu\sigma}k_2^\nu + F_4(k_1^2,k_2^2)k_1^\rho k_2^\sigma k_1^\mu G^1_{\mu\rho}G^2_{\nu\rho}k_2^\nu\Big], \end{aligned} \tag{10}$$

where

$$P_{\rho\sigma} \equiv g_{\rho\sigma} - \frac{P_\rho P_\sigma}{m^2}, \tag{11}$$

for a resonance with mass m and momentum P_μ. Here ϵ_ρ and $\epsilon_{\rho\sigma}$ are the polarization vector and tensor for a vector or tensor resonance, and satisfy the relations

$$\begin{aligned} \sum_\epsilon \epsilon_\rho\epsilon_\sigma \;&=\; g_{\rho\sigma} - \frac{P_\rho P_\sigma}{m^2} \quad or \\ \sum_\epsilon \epsilon_{\rho\sigma}\epsilon_{\rho'\sigma'} \;&=\; \frac{1}{2}(P_{\rho\rho'}P_{\sigma\sigma'} + P_{\rho\sigma'}P_{\sigma\rho'}) - \frac{1}{3}P_{\rho\sigma}P_{\rho'\sigma'}. \end{aligned} \tag{12}$$

The resonance R could be $q\bar{q}$, glueball or mixture of both. The form factors $F_i(k_1^2,k_2^2)$ depend on the composition of R. This will be discussed below. We adopt the normalization convention that $F_1[J^{PC}](0,0) = 1$. Also, we use the shorthand for on-shell form factors $F_i[J^{PC}](0,0) = F_i[J^{PC}]$. Note that for $k_1^2 = k_2^2 = 0$, $< g^a g^a | 1^{++} >= 0$ as it must by Furry's theorem. The constants A_{JPC} in Eqs. 7-10 are dependent on the coupling between gluons and the resonance constituents, the wavefunction of the resonance and its mass (specific examples for $q\bar{q}$ are given below in eq. (24)).

After summing over the color index of the final two gluon state, the general expression for the total width of a resonance R decaying into two real gluons[5] is

$$\Gamma(R \to gg) = \frac{1}{(2J+1)} \frac{1}{2m\pi} \sum_{\epsilon_1, \epsilon_2} | < gg|R > |^2, \tag{13}$$

where $< gg|R >$ is the coefficient of δ^{ab} in eqs. 7-10. For example in the case $R = 0^{++}$, using the matrix element $< gg|0^{++} >$ given by eq. 7, and summing over gluon polarizations $\epsilon_{1,2}$, gives

$$\begin{aligned} \sum_{\epsilon_{1,2}} | < gg|0^{++} > |^2 &= \frac{1}{3}|A_{O^{++}}|^2 \Big[(k_1 \cdot k_2)^2 g_{\mu\mu'} g_{\nu\nu'} + k_1^{\mu} k_1^{\nu} k_2^{\mu'} k_2^{\nu'} \\ &\quad + k_1^{\mu'} k_1^{\nu'} k_2^{\mu} k_2^{\nu} - 2 k_1 \cdot k_2 (g_{\mu\mu'} k_1^{\nu} k_2^{\nu'} + g_{\nu\nu'} k_1^{\mu'} k_2^{\mu}) \\ &\quad + k_1 \cdot k_2 (g_{\mu\nu} k_1^{\mu'} k_2^{\nu'} + g_{\mu'\nu'} k_1^{\mu} k_2^{\nu}) \Big] P_{\mu\nu} P_{\mu'\nu'} \\ &= \frac{3}{8} m^4 |A_{O^{++}}|^2, \end{aligned} \tag{14}$$

which leads to

$$\Gamma(R_{0^{++}}) = \frac{3m^3}{16\pi} |A_{0^{++}}|^2. \tag{15}$$

The decay widths for the other states are obtained by the same procedure and read

$$\Gamma(R_{0^{-+}}) = \frac{2m^3}{\pi} |A_{0^{-+}}|^2, \tag{16}$$

and

$$\Gamma(R_{2^{++}}) = \frac{m^3}{20\pi} |A_{2^{++}}|^2 \left(1 + \frac{m^4}{12} F_2^2[2^{++}] \right). \tag{17}$$

While these are nominally two-gluon widths, when the scale of resolution of the gluons is taken large enough (see ref.[18] and below) they become the total gluonic widths. Note also that since $A_{1^{++}}$ cannot be fixed in this way, we focus primarily on 0^{++}, 2^{++}, and 0^{-+} states. We return to the axial mesons in sec. 3.4

Given the form factors appearing in $< g^a g^b | R >$ and the NRQM-pQCD formula (eq. 5) for $< Q\bar{Q}_V | \gamma g^a g^b >$, the integral $\int d^4k \frac{1}{k_1^2 k_2^2} < Q\bar{Q}_V | \gamma g^a g^b >< g^a g^b | R >$ can be carried out, thus determining the $H_J(x)$'s appearing in eq. (1). The analysis of ref.[18] assumed that the relative size of the on-shell form factors and their dependence on gluon virtualities is universal, for heavy and light $q\bar{q}$ mesons and for glueballs. There is no general reason why this should be the case. For example, we know from form factors of electromagnetic and weak currents that some aspects of form factors are universal [4] while other aspects such as the relative magnitude of the nucleon on-shell form factors depend on detailed structure of the bound state, in particular the constituent quark magnetic moments. The next sections describe the information we presently have on the $< gg|R >$ form factors.

2.2 $< gg|R >$ form factors

A particular example for the $< gg|R >$ form factors is the case of $R = q\bar{q}$, where the quantity $< gg|R >$ has been modeled[19, 21, 22] as a QCD analogue[24] of the two photon coupling to positronium[25]. In the NRQM approximation[22]

$$< gg|0^{++}(q\bar{q}) >= c'\sqrt{\frac{1}{6}} \left[G^a_{\mu\nu} G^a_{\mu\nu} (m^2 + k_1 \cdot k_2) - 2k_1^\nu G^a_{\mu\nu} G^a_{\mu\rho} k_2^\rho \right] / (k_1 \cdot k_2)^2, \tag{18}$$

$$< gg|1^{++}(q\bar{q}) >= c' m \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} \epsilon_\sigma \left[G^1_{\mu\nu} G^2_{\rho\lambda} k_2^\lambda + G^2_{\mu\nu} G^1_{\rho\lambda} k_1^\lambda \right] / (k_1 \cdot k_2)^2, \tag{19}$$

and

$$< gg|2^{++}(q\bar{q}) >= c'\sqrt{2} m^2 G^a_{\mu\rho} G^a_{\nu\rho} e^{\mu\nu} / (k_1 \cdot k_2)^2, \tag{20}$$

[4] For instance, the leading Q^2 dependence of the nucleon and meson electromagnetic form factors depends only on the number of valence constituents[23].

where the constants c' are proportional to the derivative of the radial wave-functions at the origin:

$$c' = g_s^2 \sqrt{\frac{1}{m^3 \pi}} R'(0). \tag{21}$$

Analogously the two gluon coupling for a 0^{-+} state is[22]

$$< gg|0^{-+}(q\bar{q}) >= c\epsilon_{\rho\sigma\mu\nu} G^a_{\rho\sigma} G^a_{\mu\nu}/k_1 \cdot k_2, \tag{22}$$

where

$$c = g_s^2 \frac{1}{4} \sqrt{\frac{1}{3m\pi}} R(0). \tag{23}$$

The above are particular models for Eqs. 7-10. To see the correspondence between the quantities $< gg|J^{PC}(q\bar{q}) >$ and Eqs. 7-10, it is convenient to note that Eq. 18 can be rewritten in the form of Eq. 7:

$$< gg|0^{++}(q\bar{q}) >= c' m^2 \sqrt{\frac{2}{3}} G^{1a}_{\alpha\mu} G^{2a}_{\alpha\nu} P_{\mu\nu}/(k_1 \cdot k_2)^2, \tag{24}$$

The constants A_{JPC} in our Eqs. 7-10 are thus related to the constants c and c' of ref.[22] (Eqs. 21 and 23 above) by

$$\begin{aligned} A_{0^{++}} = A_{2^{++}} = \sqrt{8} m A_{1^{++}} &= \frac{4\sqrt{2}c'}{m^2} \\ A_{0^{-+}} &= \frac{2c}{m^2}, \end{aligned} \tag{25}$$

and the matching implies that the only non-zero form factors for $< gg|q\bar{q} >$ in the NRQM are

$$F_1[0^{++}] = F_1[2^{++}] = F_1[1^{++}] = \frac{m^4}{4(k_1 \cdot k_2)^2}, \tag{26}$$

and[32]

$$F_1[0^{-+}] = \frac{m^2}{2k_1 \cdot k_2}. \tag{27}$$

Substituting Eq. 25 into Eqs. 15-17, gives

$$\Gamma((q\bar{q})_{0^{++}}) = 96 \frac{\alpha_s^2}{m^4} |R'(0)|^2, \tag{28}$$

$$\Gamma((q\bar{q})_{2^{++}}) = \frac{128}{5}\frac{\alpha_s^2}{m^4}|R'(0)|^2, \tag{29}$$

and

$$\Gamma((q\bar{q})_{0^{-+}}) = \frac{8}{3}\frac{\alpha_s^2}{m^2}|R(0)|^2, \tag{30}$$

which agree with those in Ref. [18].

The gluon structure appearing in the form factors for the scalar and tensor NRQM mesons (e.g., $G^a_{\alpha\mu}G^a_{\alpha\nu}P_{\mu\nu}$) has been widely employed also for scalar and tensor glueballs[26]. It can be considered a natural relativistic generalization of TE mode glueballs in a cavity approximation such as the MIT bag model[27]:

$$\psi(G_{J^{++}}) = \langle 1\alpha; 1\beta | J, \alpha+\beta\rangle (\vec{\epsilon_1} \times \hat{\vec{k_1}})^{(\alpha)} (\vec{\epsilon_2} \times \hat{\vec{k_2}})^{(\beta)} \phi(r) \tag{31}$$

where $\phi(r)$ is a radial wavefunction, and the superscripts α ,β specify the projection of the angular momenta along $\hat{z}$. The relativistic generalization adopted in ref. [26] produces

$$\psi(G_{0^{++}}) = \frac{1}{\sqrt{3}} P_{\mu\nu} \frac{G^{1a}_{\mu\rho}G^{2a}_{\nu\rho}}{\sqrt{8}k_1 \cdot k_2}\phi(x) \tag{32}$$

for the scalar and

$$\psi(G_{2^{++}}) = \epsilon_{\mu\nu} \frac{G^{1a}_{\mu\rho}G^{2a}_{\nu\rho}}{\sqrt{8}k_1 \cdot k_2}\phi(x) \tag{33}$$

for the tensor states, where $P_{\mu\nu}$ and $\epsilon_{\mu\nu}$ are defined in eqs. 11 and 12. Note that in cavity approximation the same function $\phi(x)$ appears for both 0^{++} and 2^{++} states, so that the relative magnitudes of their form factors are fixed. The resulting matrix elements for the two gluon couplings of the glueballs are

$$< gg|G(J^{PC}) >= Af(k_1^2, k_2^2)\mathcal{P}^J_{\mu\nu}\frac{G^{1a}_{\mu\rho}G^{2a}_{\nu\rho}}{k_1 \cdot k_2} \tag{34}$$

where $\mathcal{P}^0_{\mu\nu} \equiv \frac{P_{\mu\nu}}{\sqrt{3}}$ and $\mathcal{P}^2_{\mu\nu} \equiv \epsilon_{\mu\nu}$. The form factor $f(k_1^2, k_2^2)$ in Eq. 34 is determined by the wavefunctions $\phi(x)$ which appears in eqs. 32 and 33.

Comparison with eqs. (7) and (10) shows that the glueball wavefunction (34) corresponds to $A_{0^{++}}F_1[0^{++}](k_1^2, k_2^2) = A_{2^{++}}F_1[2^{++}](k_1^2, k_2^2)$; the remaining form factors vanish. Since this relation between 0^{++} and 2^{++} form factors is the same as in the NRQM, both models give the same result for the ratio of the 2^{++} and 0^{++} widths. Thus in the limit that the masses of the scalar and tensor states are equal, independently of whether they are a pair of (NRQM) $q\bar{q}$ [24, 25, 28] or (cavity approximation) glueball states,

$$\frac{\Gamma(R_{2^{++}})}{\Gamma(R_{0^{++}})} = \frac{4}{15}. \tag{35}$$

The large mass gap between the 0^{++} glueball candidates $f_0(1500)$ and $f_0(1700)$, and the 2^{++} candidate $\xi(2230)$ (assuming $\xi(2230)$ has $J = 2$) prevents immediate application of eq. 35. However since these f_0's have widths 100-150 MeV and the $\xi(2230)$ width is $\sim$ 20 MeV (see sec. 3.3) – either f_0 is compatible with (35). The presence of 0^{++} $q\bar{q}$ states in the vicinity of these f_0's also complicates the situation (see sec. 6).

For a light $q\bar{q}$ system, Eq. 35 will be modified by relativistic effects[28], which increases the ratio 4/15 to around $\frac{1}{2}$. While there are no $R \to gg$ data available, one could relate the width for $R \to gg$ to that for the $R \to \gamma\gamma$ at the tree level. The data for $f_0(1300) \to \gamma\gamma$ and $f_2(1270) \to \gamma\gamma$ are consistent with the result here. However, the relativistic effects on the loop integral $x|H(x)|^2$ remain to be investigated.

2.3 Virtuality Dependence of Glueball Form Factors

The analysis of J/ψ radiative decays in ref.[18] implicitly assumed that the relative size of the on-shell form factors and also their dependence on gluon virtualities are universal for $q\bar{q}$ mesons and glueballs. Our investigation of the previous section showed that the first assumption may be reasonable.

For the $< gg|R >$ form factors the situation is more complicated than for electromagnetic form factors, because the gluon virtualities can vary independently. That is, the form factors here are functions of two variables, constrained by the requirement of being even under interchange of the gluon momenta (from Bose statistics). We can further constrain the form factors by power counting arguments. Replace the variables k_1^2 and k_2^2 by $k_1 \cdot k_2$ and the dimensionless ratio $z \equiv \frac{(k_1 - k_2)\cdot(k_1+k_2)}{k_1 \cdot k_2}$. When R is an $L = 0$ bound state of two constituents, the leading large $k_1 \cdot k_2$ behavior of $F_1(k_1^2, k_2^2)$ is $\frac{1}{(k_1 \cdot k_2)} f(z)$. The $F_i(k_1^2, k_2^2)$ entering $< gg|R >$ with additional factors of k_i^μ have correspondingly more rapid falloff, like F_2 compared to F_1 for the case of electromagnetic form factors. For $L = 1$ systems one expects an additional $\mu^2/k_1 \cdot k_2$ suppression, where μ is a scale reflecting the variation of the wavefuntion at the origin. This scaling behavior is manifested by the NRQM results eqs. 18 - 22.

Neglect of higher twist corrections to the leading form factors and neglect of those form factors whose leading dependence falls more rapidly, can be expected to give corrections to the $H_J(x)$'s of order m_R^2/M_V^2 compared to the leading terms. Since for our application this is a small quantity, we neglect these corrections. As we saw in sec. 2.2, in the NRQM $f(z) = const$. The effect of possible corrections to constancy of $f(z)$, and overall scaling behavior which differs from the NRQM, is presently under study.

2.4 Higher Order Corrections and Scale Dependence

For a resonance which couples to the two-gluon intermediate state, corrections to the above formalism involve one additional gluon loop and thus should be of order $O(\alpha_s/4\pi)$ in the amplitude. For heavy $Q\bar{Q}$ mesons the wavefunction can be treated perturbatively and it is straightforward to make a systematic expansion in the coupling constant[24, 29]. In this context it is sensible to distinguish between the components of the wavefunction in which the $Q\bar{Q}$ are in a "color singlet" or "color octet" state[30]. However for the light $q\bar{q}$ mesons and glueballs of interest, defining a perturbative expansion and the relation between "constituent" and "current" partons are more subtle and we do not undertake this here. Suffice it to say that the concept of "color singlet" versus "color octet" components of the wavefunction does not have an *a priori* well-defined intuitive meaning as for the heavy quark system. The issue of composition is a scale-dependent question, as it is for the nucleon.

The Altarelli-Parisi evolution of the parton distribution functions is a clear illustration of this point. In principle the same is true for the heavy quark system, however the quark mass gives a natural scale in that case.

Our treatment in previous sections implicitly made use of an effective Lagrangian approach to the problem. By working with $b_{rad}(Q\bar{Q}_V \to \gamma + R)$, i.e., dividing $\Gamma(Q\bar{Q}_V \to \gamma + R)$ by $\Gamma(Q\bar{Q}_V \to \gamma + X)$ also computed in leading order perturbation theory, one removes the dependence on the effective strong coupling at the heavy quark vertices. Similarly, quoting the result in terms of $br(R \to gg)$ removes at leading order the sensitivity to the scale dependence of the definition of the R wavefunction. Of course, the concept of "gluonic width" of a $q\bar{q}$ resonance necessarily has an intrinsic scale dependence – as one goes to shorter and shorter distance scale, contributions from the parton sea invalidate simple valence intuition. Without a careful treatment of next-to-leading order corrections, we cannot specify the correct scale for α_s appearing in the estimate of eq.(1) for the gluonic branching fraction of $q\bar{q}$ mesons. For this reason, we can make only qualitative use of the gluonic branching fractions that we extract for $q\bar{q}$ mesons. However when the branching fraction of a state is found to be $\gtrsim 1/2$, indicating that the state has a significant gluon component, the sensitivity on α_s is a higher order correction and we *can* make quantitative use of the $br(R \to gg)$ that we extract from the data.

It follows from the above discussion that we cannot expect a trivial relationship between the form factors for the amplitude $< gg|R >$, and those for $< R|\gamma\gamma >$. In principle the latter can be measured as a function of photon virtualities in an e^+e^- collider. For heavy $Q\bar{Q}$ resonances such as the χ_c states, these amplitudes are identical except for the value of the overall coefficient $A_{J^{PC}}$. To obtain $< R|\gamma\gamma >$ from $< gg|R >$, substitute $g_s \to e_Q$ and remove the color factor. At leading order this gives

$$\Gamma(R \to \gamma\gamma) = \frac{9e_Q^4}{2}(\frac{\alpha}{\alpha_s})^2\Gamma(R \to gg). \tag{36}$$

We test the validity of eq. (36) for light $q\bar{q}$ resonances in section 5 by applying it to the known $q\bar{q}$ states $f_2(1270)$ and $f_2(1525)$. We then extend it to other examples. Insisting on the naive relation (36) allows one to extract an effective value of α_s. Doing so for several $q\bar{q}$ resonances gives some idea of the sensitivity of $br(R \to gg)$ to scale.

It might be the case that the dynamics of the form factors, i.e., the functional dependence of the F_i's on k_1^2 and k_2^2 and their relative normalization at the on-shell point, corresponds more accurately than does their overall normalization, in going from $< R|\gamma\gamma >$ to $< gg|R >$[5]. This could in principle be tested by measuring the off-shell form factors in a $\gamma\gamma$ collider and using this dependence to predict the $H_J(x)$'s appearing in eq. (1). Once $b_{rad}(Q\bar{Q}_V \to \gamma + R)$ is measured for both J/ψ and Υ radiative decay to a given $q\bar{q}$ meson, one can infer H_J at two values of x. With several related $q\bar{q}$ mesons of different masses, such as $f_2(1270)$ and $f_2(1520)$, this will give a number of points in $H_J(x)$.

3 Constraints From Radiative Quarkonium Decay

3.1 J^P dependence of $Q\bar{Q}_V \to \gamma R_{J^P}$

The loop integral in eq. (4) determines the function $H_J(x)$ appearing in (1). For the NRQM wavefunctions these integrals have been evaluated[19, 32] in analytical form and are recorded in the appendix for convenience. Readers interested in the derivation of the analytic expressions are referred to those papers. The relevant functions $x|H_J(x)|^2$ are shown in Fig. 1 for $J^{PC} = 0^{++}$, 2^{++} and 0^{-+}. The corresponding c_R's in Eq. 1 are:

$$c_R = \begin{cases} 1 & J^{PC} = 0^{-+} \\ \frac{2}{3} & J^{PC} = 0^{++} \\ \frac{5}{2} & J^{PC} = 2^{++}. \end{cases} \tag{37}$$

In the x regime of immediate interest, $x \sim 0.5 - 0.75$, we note from Fig. 1 that $\frac{x|H_J|^2}{30-45} \sim O(1)$. This enables us to manipulate the CF expression,

[5] E.g., for $q\bar{q}$ states, the overall normalization of $< gg|R >$ contains a factor α_s and is necessarily scale dependent.

eq. (1), into a scaled form that exhibits the phenomenological implications immediately. Specifically, for scalar mesons

$$10^3 br(J/\psi \to \gamma 0^{++}) = (\frac{m}{1.5\ GeV})(\frac{\Gamma_{R\to gg}}{96\ MeV})\frac{x|H_S(x)|^2}{35}. \tag{38}$$

This is to be compared with the analogous formula for a tensor meson:

$$10^3 br(J/\psi \to \gamma 2^{++}) = (\frac{m}{1.5\ GeV})(\frac{\Gamma_{R\to gg}}{26\ MeV})\frac{x|H_T(x)|^2}{34}. \tag{39}$$

For pseudoscalars we find:

$$10^3 br(J/\psi \to \gamma 0^{-+}) = (\frac{m}{1.5\ GeV})(\frac{\Gamma_{R\to gg}}{50\ MeV})\frac{x|H_{PS}(x)|^2}{45}. \tag{40}$$

Having scaled the expressions this way, because $\frac{x|H_J|^2}{30-45} \sim O(1)$ in the x range relevant for production of 1.3 - 2.2 GeV states (see fig. 1), we see immediately that the magnitudes of the branching ratios are driven by the denominators 96 and 26 MeV for 0^{++} and 2^{++}, and 50 MeV for 0^{-+}. Thus if a state R_J is produced in $J/\psi \to \gamma X$ at $O(10^{-3})$ then $\Gamma(R_J \to gg)$ will typically be of the order 100 MeV for 0^{++}, $O(25\ MeV)$ for 2^{++}, and $O(50\ MeV)$ for 0^{-+}.

This immediately shows why the 2^{++} $q\bar{q}$ states are prominent: A 2^{++} state with a total width of $O(100\ MeV)$ (typical for 2^{++} $q\bar{q}$'s in this mass range[2, 31]) will be easily visible in $J/\psi \to \gamma 2^{++}$ with branching fraction $O(10^{-3})$, while remaining consistent with

$$br(R[Q\bar{Q}] \to gg) = 0(\alpha_s^2) \simeq 0.1 - 0.2. \tag{41}$$

Eqs. 38 - 40 not only indicate which $q\bar{q}$ states will be prominent in $J/\psi \to \gamma R$, but they also help to resolve an old paradox concerning 0^{++} production. It was recognised early on that when the gluons in the absorbtive part of $J/\psi \to \gamma gg$ are classified according to their J^{PC}, the partial wave with 2^{++} was predicted to dominate. The waves with 0^{-+} and 0^{++} were also predicted to be significant and of comparable strength to one another [20]. When extended to include the dispersive part[18, 19] the 0^{++} was predicted to be prominent over a considerable part of the kinematic region of interest. States with $J \geq 3$

were predicted to have very small rate in this process. Experimentally, all but one of these appeared to be satisfied. There are clear resonant signals in 2^{++} and 0^{-+}, and no unambiguous signals have been seen with $J \geq 3$. However no 0^{++} signal had been isolated.

¿From our relations above, we see that for a 0^{++} to be produced at the 10^{-3} level in J/ψ radiative decay it must either have a large gluonic content and width $O(100)$ MeV or, if it is a $q\bar{q}$ meson, it must have a very large width, $\gtrsim 500$ MeV. Taking this into account, along with the following points, the puzzle of the absence of 0^{++} signal has been resolved:

(i): The width of $^3P_0$ $q\bar{q}$ is predicted to be $\sim$ 500 MeV[2, 31]. Thus production at the level $br(J/\psi \to \gamma(gg)_{0^+} \sim 10^{-3})$ is consistent with $br(R \to gg) = 0(\alpha_s^2) \simeq 0.1 - 0.2$, but the $\sim$ 500 MeV wide signal is smeared over a large kinematic (x) range.

(ii): The ~ 100 MeV wide $f_0(1500)$ signal seen in $J/\psi \to \gamma 4\pi$ was originally misidentified as 0^{-+}, but is now understood to be 0^{++}[3].

(iii): The $f_J(1710)$ which was originally believed to be $J = 2$ may contain a contribution with $J = 0$[3, 5].

3.2 Flavor Mixing and the $f_2(1270)$ and $f_2(1520)$ $q\bar{q}$ States

We can test this formalism by applying it first to the the well known quarkonium states $f_2(1270)$ and $f_2(1525)$. The above formulae have been derived for the case that the produced meson $R(q_i\bar{q}_i)$ contains a single flavour, so we begin by considering what changes occur for a state of mixed flavour. We shall see that $br(J/\psi \to \gamma R)$ and $\Gamma(R \to gg)$ depend on the mass and flavour of R, but in a common way such that $br(R \to gg)$ is universal, as summarized in eqs.38-40.

For a general $q\bar{q}$ resonance

$$R \equiv cos\phi|n\bar{n}\rangle + sin\phi|s\bar{s}\rangle, \tag{42}$$

where $n\bar{n} \equiv \frac{u\bar{u}+d\bar{d}}{\sqrt{2}}$. Allowing for flavour symmetry breaking

$$\langle gg|s\bar{s}\rangle \equiv r_s^2 \langle gg|d\bar{d}\rangle. \tag{43}$$

Thus

$$\langle gg|R\rangle = (\sqrt{2}cos\phi + r_s^2 sin\phi)\langle gg|d\bar{d}\rangle, \tag{44}$$

so

$$\Gamma(R \to gg) \equiv (\sqrt{2}cos\phi + r_s^2 sin\phi)^2 \Gamma(R(d\bar{d}) \to gg) \tag{45}$$

and similarly

$$\Gamma(V \to \gamma R) = \Gamma(V \to \gamma R(d\bar{d}))(\sqrt{2}cos\phi + r_s^2 sin\phi)^2. \tag{46}$$

Evidently, the flavour factors cancel out in derivation of the expressions of the previous sections and so apply immediately to states R of arbitrary flavour mixings. We can illustrate this with the $^3P_2$ states $f_2(1270)$ and $f_2(1520)$, for which $cos\phi \sim 1$ and 0 respectively. From eq. 45 we have

$$\Gamma(f_2(n\bar{n})(1270) \to gg)/\Gamma(f_2(d\bar{d})(1270) \to gg) = 2 \tag{47}$$

and

$$\Gamma(f_2(s\bar{s})(1520) \to gg)/\Gamma(f_2(d\bar{d})(1520) \to gg) = r_s^2. \tag{48}$$

If $q\bar{q} \to gg$ is flavour blind we expect $r_s^2 \sim 1$.

To confront these equations with data we use the measured radiative branching ratios[5]

$$10^3 \times br(J/\psi \to \gamma f_2(1270)) = 1.4 \pm 0.14 \tag{49}$$

and

$$10^3 \times br(J/\psi \to \gamma f_2(1520)) = 0.63 \pm 0.1. \tag{50}$$

¿From eq. (39), we have

$$\Gamma(1270 \to gg) = 41 \pm 7 \ MeV. \tag{51}$$

and

$$\Gamma(1520 \to gg) = 17 \pm 2 \ MeV. \tag{52}$$

Combining these results with the measured widths,

$$\Gamma_{tot}(1270) = 185 \pm 20 \ MeV; \quad \Gamma_{tot}(1520) = 76 \pm 10 \ MeV, \tag{53}$$

we find

$$br(f_2(1270) \to gg) \simeq br(f_2(1520) \to gg) = 0.22 \tag{54}$$

which are as expected for established $q\bar{q}$ states, see eq.(1) and ref. [18]. Inter alia this supports the idea that glueball mixing is not prominent in the 2^{++} channel at these masses[2].

If the dependence on mass is weak in going from 1270 to 1520 MeV, eqs.47 and 48 imply

$$\frac{\Gamma(f_2(1270) \to gg)}{\Gamma(f_2(1520) \to gg)} = \frac{2}{r_s^2}. \tag{55}$$

Inserting the widths from eqs. 51 and 52 we see that $r_s \simeq 0.8 - 1$, and thus $q\bar{q}$ flavor can be ignored to first approximation in this analysis. We will exploit this flavor independence in a later section (4.1) to probe the structure of wavefunctions for potential $q\bar{q}$ mesons.

3.3 Radiative Upsilon Decay

No peaks are seen in the photon energy spectrum in inclusive $\Upsilon \to \gamma X$ at a branching fraction sensitivity of about 10^{-4}[5]. The following analysis suggests that with only a factor of a few improvement in sensitivity, many interesting states should become evident. We could use data on $\Upsilon \to \gamma R$ in two ways. Firstly, for $R \equiv c\bar{c}$, pQCD-NRQM predictions should be reliable. Testing those predictions tests the underlying assumption of this methodology: that pQCD provides an adequate description of the $< Q\bar{Q}_V|\gamma gg >$ amplitude. Secondly, production of a given resonance in $\Upsilon \to \gamma R_J$ depends on $H_J(x)$ at x much closer to 1. This allows a more detailed examination of the form factors, as well as probing the $x \to 1$ region where resummation of perturbation theory may be required for the 0^{++} and 2^{++} cases.

Let us begin by considering $\Upsilon \rightarrow \gamma\chi_c$, where the $Q\bar{Q}$ bound states are rather well understood[19]. The branching ratio is

$$br(\Upsilon \rightarrow \gamma + R_c) = c_J(\frac{4}{5}\frac{\alpha}{\alpha_s})\Gamma_{R\rightarrow gg}\frac{x|H_J(x)|^2}{8\pi(\pi^2-9)}\frac{m}{M^2}. \tag{56}$$

where $c_J = 1(\eta_c),\ \frac{2}{3}(\chi_c^0)$ and $\frac{5}{2}(\chi_c^2)$. In all these examples $x \equiv 1 - m_R^2/M_\Upsilon^2 \sim 0.9$ so from fig. 1 we have $x|H|^2 = 54(\eta_c),\ 32(\chi_c^0),\ 37(\chi_c^2)$. If we use $\alpha_s(m_b) \sim 0.18$ and include the one loop corrections from [33], we find

$$br(\Upsilon \rightarrow \gamma gg) \equiv \frac{4\alpha}{5\alpha_s}(1 - 2.6\frac{\alpha_s}{\pi}) \sim 2.8\%.$$

Identifying $\Gamma(c\bar{c} \rightarrow gg)$ with $\Gamma(c\bar{c} \rightarrow \text{light hadrons})$ implies

$$br(\Upsilon \rightarrow \gamma\chi_2) \sim br(\Upsilon \rightarrow \gamma\chi_0) \sim 0.9 \times 10^{-5}$$

and

$$br(\Upsilon \rightarrow \gamma\eta_c) \sim (2.3 \pm 0.9) \times 10^{-5}.$$

Although these predicted branching ratios are small, the photons are in a region of phase space where there is little background, so a relatively short period of dedicated running at a B factory should be adequate to observe these modes. Precision data on these transitions could both validate the pQCD analysis and give insights into higher order effects including the role of colour octet components in the χ wavefunctions.

Data on $\Upsilon \rightarrow \gamma f_{0,2}(1270 - 1700)$ may also be obtained, replacing the present upper limits $\leq 10^{-4}$[5]. The kinematics here are $x \sim 0.97$. In this region $|H_S(x)|^2$ and $|H_T(x)|^2$ are dominated by $ln(1-x)$ divergences, and the leading order pQCD predictions become unreliable. We urge that studies of pQCD resummation be made in order to analyze this process and make predictions. Data on these processes could be used both to extract $|H_J(x)|^2$ for phenomenological use as in sec. 4, and also permit detailed testing of pQCD resummation techniques. The qualitatively different behavior of the $x \rightarrow 1$ limits of $H_J(x)$ for the 0^{-+} and $0, 2^{++}$ cases can also be exploited to this end.

3.4 1^{++} States

Before turning to our main topic of glueball candidates, we discuss briefly the possibility of applying this formalism to 1^{++} mesons. Since for them we cannot normalize the $< gg|R >$ amplitudes using the procedure of sec. 3.1 which lead to eqs. (15 - 17) it is not certain that this is possible. However in the spirit of an effective lagrangian approach it might be appropriate to consider that gluons have an effective mass, so that the amplitude $< gg|R >$ need not vanish at the on-shell point when one of the gluons is longitudinal. Making the further assumption that the pQCD-NRQM approximation of ref. [29] gives an adequate description of the 1^{++} total width, with an adjustable overall normalization, one can obtain a relation of the form of eq. (1)[18]. Substituting for $H_1(x)$ leads to the scaled formula

$$10^3 br(J/\psi \to \gamma 1^{++}) = (\frac{m}{1.45\ GeV})(\frac{\Gamma_{R\to gg}}{12\ MeV})\frac{x|H_1(x)|^2}{30} \tag{57}$$

where in this application $\Gamma(R \to gg)$ is the total direct coupling to gluons, not literally the coupling to two massless gluons.

It is interesting to apply the above relation to the $f_1(1285)$, $f_1(1420)$ and $f_1(1530)$ states (see also ref.[19]). Only two isoscalar mesons can be accomodated in a quarkonium nonet and there has been considerable discussion as to which of the three axial states is the odd one out (and, if it exists, what its nature is). There has been no confirmed sign of $f_1(1530)$ in J/ψ radiative decay whereas the $f_1(1420)$ and $f_1(1285)$ are both seen. Their branching ratios are respectively[5]

$$br(J/\psi \to \gamma f_1(1285)) = (0.65 \pm 0.10) \times 10^{-3} \tag{58}$$

and

$$br(J/\psi \to \gamma f_1(1420)) \times br(f_1(1420) \to K\bar{K}\pi) = (0.83 \pm 0.15) \times 10^{-3}. \tag{59}$$

Inserting these values into eq. 57 together with $\Gamma_{tot}(f_1(1285)) = 24 \pm 3$ MeV and $\Gamma_{tot}(f_1(1420)) = 52 \pm 4$ MeV give

$$br(f_1(1285) \to gg) = 0.34 \pm 0.05 \tag{60}$$

and

$$br(f_1(1420) \to gg) = (0.19 \pm 0.04)/(br(f_1(1420) \to K\bar{K}\pi)). \qquad (61)$$

If we now input $br(f_1(1420) \to K\bar{K}\pi) \sim 0.67$[34] we find

$$br(f_1(1420) \to gg) \sim 0.3. \qquad (62)$$

Within the uncertainties of applying these ideas to low masses (e.g.$f_1(1285)$ has a low width due to phase space suppression of KK^*) and the ill defined branching ratio to $K\bar{K}\pi$ for the $f_1(1420)$, these results are not inconsistent with the accepted $q\bar{q}$ interpretation of the $f_1(1285)$ and support also the quarkonium interpretation of $f_1(1420)$ (unless the $br(K\bar{K}\pi)$ should turn out to be much overestimated).

The ratio

$$\frac{br(J/\psi \to \gamma f_1(1420))}{br(J/\psi \to \gamma f_1(1285))} \sim 1.9 \qquad (63)$$

is consistent with the quarkonium mixing arising from a quadratic mass formula for the axial nonet[34]

$$f_1(1285) = 0.94|n\bar{n}\rangle - 0.35|s\bar{s}\rangle \equiv 0.57|1\rangle + 0.83|8\rangle \qquad (64)$$

$$f_1(1420) = 0.35|n\bar{n}\rangle + 0.94|s\bar{s}\rangle. \equiv 0.83|1\rangle - 0.57|8\rangle \qquad (65)$$

Recent data from BES[35] have large error bars but are consistent with the older data for $f_1(1420)$; they obtain $br(J/\psi \to \gamma f_1(1420)) \times br(f_1(1420) \to K\bar{K}\pi) = (0.76 + 0.46 - 0.18) \times 10^{-3}$, $\Gamma = 59 \pm 5$ MeV. They do not see any $f_1(1285)$ but this may not be surprising since they are looking in the $K\bar{K}\pi$ mode. They also report a signal $f_1(1497)$, $\Gamma = 44 \pm 7$ MeV and $br(J/\psi \to \gamma f_1(1497)) \times br(f_1(1497) \to K\bar{K}\pi) = (0.52 \pm 0.23) \times 10^{-3}$. This state's parameters are also consistent with those expected for a quarkonium as long as $br(f_1(1497) \to K\bar{K}\pi) \geq 0.5$.

The axial mesons are currently an enigma. There are three candidates where the quark model would require only two. The lattice predicts the lightest 1^{++} glueball to be at ~ 4 GeV[15]. It is noticeable that no single experiment sees all three and one should be cautious as to whether there are indeed three genuine states. We urge that BES, in particular, seek three f_1 signals or place limits against them in order to help clarify the above analysis. In any event, more detailed theoretical work, specifically formalizing the effective lagrangian treatment of the problem, is warranted in order to relate to the production of axial mesons in $\gamma\gamma^*$ and to provide a more solid foundation to the theoretical analysis after the experimental situation becomes clear.

4 Glueball Candidates

4.1 $f_0(1500)$

We look first at the established scalar meson, $f_0(1500)$. As we shall see below, at present there are discrepancies between the values of $br(J/\psi \to \gamma f_0(1500))$ as determined from various experimental analyses.

The analysis of ref. [3] gives $br(J/\psi \to \gamma f_0(1500) \to \gamma\sigma\sigma) = (5.7 \pm 0.8) \times 10^{-4}$ with an overall $\pm 15\%$ normalisation uncertainty. The analysis of ref.[36, 37] implies that the $\sigma\sigma$ mode is at most 50% branching ratio and so we infer

$$br(J/\psi \to \gamma f_0(1500)) \geq (1.15 \pm 0.15) \times 10^{-3} \pm 15\%.$$

In this case, with $\Gamma_{tot}(f_0(1500)) = 120 \pm 20$ MeV, if we add errors in quadrature and use the central value, eq. 38 implies that

$$br(f_0(1500) \to gg) \geq 0.9 \pm 0.2. \tag{66}$$

This is significantly larger than the $O(\alpha_s^2)$ which would be expected for a pure $q\bar{q}$ system, and supports this state as a glueball candidate.

On the other hand BES has recently reported[38] $br(J/\psi \to \gamma f_0(1500) \to \gamma\pi^o\pi^o) = 3 - 5 \times 10^{-5}$. Landua[38] combines this with the Crystal Barrel data on $br(f_0(1500) \to \pi\pi)$ to get

$$br(J/\psi \to \gamma f_0(1500)) = (0.4 - 0.6) \times 10^{-3}.$$

Thus via eq. (38),

$$br(f_0(1500) \to gg) = 0.3 - 0.5. \tag{67}$$

The interpretation of this state cannot be settled until the experimental situation clarifies. An order of magnitude increase in statistics for $J/\psi \to \gamma\pi\pi\pi\pi$ will enable extension of the analysis shown in fig.2 of ref.[3], and $J/\psi \to \gamma\pi\pi$ likewise needs to be improved. The neutral channel $J/\psi \to \gamma\pi^o\pi^o\pi^o\pi^o$ is particularly advantageous here as it is free from $\rho\rho$ contamination and so can help to improve the quantification of $br(f_0(1500) \to \sigma\sigma)$. These should be high priorities at a τ-Charm Factory.

We shall return to the interpretation of the $f_0(1500)$ in section 6.

4.2 $f_J(1710)$

The case of $f_J(1710)$ is particularly interesting and the conclusions depend critically on whether $J = 0$[3, 39] or $J = 2$[5, 40, 41]. It has been observed most clearly in radiative J/ψ decay in the $K\bar{K}$ mode[40], with evidence also in the 4π mode[3]. Recently BES has reported seeing both $J = 0$ and $J = 2$ states in this region. We discuss the various measurements in turn.

In the $K\bar{K}$ channel, $br(J/\psi \to \gamma f_J(1710) \to \gamma K\bar{K}) = (0.97 \pm 0.12) \times 10^{-3}$[5]. Assuming first that $f_J(1710)$ is a single state with $J = 2$, we use eq. (39):

$$10^3 br(J/\psi \to \gamma f_2(1710)) = (\frac{m}{1.5\ GeV})(\frac{\Gamma_{R\to gg}}{26\ MeV})\frac{x|H_T(x)|^2}{34},$$

which implies

$$\Gamma(f_2(1710) \to gg) = \frac{(22 \pm 3)\ MeV}{br(f_2(1710) \to K\bar{K})}. \tag{68}$$

No comparable signal has been seen in any other channel in $J/\psi \to \gamma f_2(1710) \to \gamma X$ and it would thus appear that $K\bar{K}$ is a major mode of any $J = 2$ object in J/ψ radiative decays (the listing of decay channels for $J/\psi \to \gamma f_J(1710) \to \gamma K\bar{K}$ in ref.[5] suggests that this mode is greater than $\sim 50\%$ of the $K\bar{K} + \pi\pi + \eta\eta$ channels together). With $\Gamma_{tot}(f_J(1710)) \sim 150$ MeV, eq. 68 and $br(f_2(1710) \to K\bar{K}) \geq 0.5$ imply

$$br(f_2(1710) \to gg) \lesssim 30\%, \tag{69}$$

which would be consistent with this state being a $q\bar{q}$.

By contrast, if $f_J(1710)$ is a single state with $J = 0$, we use eq. (38):

$$10^3 br(J/\psi \to \gamma 0^{++}) = (\frac{m}{1.5\ GeV})(\frac{\Gamma_{R\to gg}}{96\ MeV})\frac{x|H_S(x)|^2}{35},$$

which implies

$$\Gamma(f_0(1710) \to gg) = \frac{(78 \pm 10)\ MeV}{br(f_0(1710) \to K\bar{K})} \tag{70}$$

and hence

$$br(f_0(1710) \to gg) \geq 0.52 \pm 0.07, \tag{71}$$

in accord with fig. 14 of ref.[18]. In this case the $f_0(1710)$ would be a strong candidate for a scalar glueball. Knowing the spin and $K\bar{K}$ branching fraction of $f_J(1710)$ is of great importance for a more detailed quantitative understanding of the composition of this state.

These questions have become central in view of new data from BEPC[42] which, for the first time, separates a $J = 2$ and $J = 0$ signal from the "$\theta(1710)$" region. They find an $f_2(1696)$ with $\Gamma = 103 \pm 18$ MeV and $br(J/\psi \to \gamma f_2) \times br(f_2 \to K^+K^-) = 2.5 \pm 0.4(10^{-4})$. They also find an $f_0(1780)$ with $\Gamma = 85 \pm 25$ MeV and $br(J/\psi \to \gamma f_0) \times br(f_0 \to K^+K^-) = 0.8 \pm 0.1(10^{-4})$. These signals are weak, $O(10^{-4})$, in contrast to the $O(10^{-3})$ reported in the earlier literature cited above. The BEPC $J = 2$ state is consistent with a (radial excited) $q\bar{q}$. Their $J = 0$ state strength appears too feeble for a glueball, unless K^+K^- is a minor decay mode.

If the BEPC data are definitive, then the possibility that $br(f_0(1780) \to K^+K^-)$ is small merits investigation. In this context we note that ref. [3] analyzes $J/\psi \to \gamma 4\pi$ and finds a signal at about 1750 MeV consistent with 0^{++}, although 2^{++} is not absolutely excluded. If interpreted as f_0, the width is $\Gamma =$

160 MeV and branching fraction in $J/\psi \to \gamma f_0 \to \gamma 4\pi$ is $(0.9 \pm 0.13) \times 10^{-3}$[3]. Thus the analysis of ref [3] indicates that there is a scalar signal in $J/\psi \to \gamma 4\pi$ at strength characteristic of gluonic states. We urge that BEPC investigate the 4π channel to see if their scalar state is visible at a level consistent with the above analysis of ref.[3].

A possible explanation of the observations, if both $f_0(1500)$ and $f_0(\text{“}1710\text{''})$ are produced at the 10^{-3} level in J/ψ radiative decay, is that both of them contain both $q\bar{q}$ and gg components [2, 43, 44, 45]. Better data, especially on the decay branching fractions and production in radiative J/ψ decay is crucial for resolving this question. We will consider complementary tests for this, through $\gamma\gamma$ production, in section 5. We examine mixing phenomenology in section 6.

4.3 $\xi(2230)$ Tensor Glueball candidate

The appearance of a narrow state $\xi(2230)$ in $J/\psi \to \gamma\pi^+\pi^-; \gamma K^+K^-$; $\gamma K_s^o K_s^o$; $\gamma p\bar{p}$ has created considerable interest[6]. In each of these channels the branching ratios are typically $br(J/\psi \to \gamma\xi) \times br(\xi \to X\bar{X}) \sim 3 \times 10^{-5}$ for each of the channels where $X \equiv \pi, K^+$ or K_s^o, and $\sim 1.5 \times 10^{-5}$ for $p\bar{p}$. In all channels the signal is consistent with $\Gamma_{tot} \sim 20\ MeV$. After allowing for associated neutral modes such as $\pi^o\pi^o$, $\eta\eta$ and $n\bar{n}$ by isospin, this gives

$$br(J/\psi \to \gamma\xi) \geq 0.1 \times 10^{-3}. \tag{72}$$

When combined with our formulae eqs. 38 and 39 this implies that $br(\xi \to gg) \geq 0.4$ for $J = 0$ and ≥ 0.15 for $J = 2$. However eq. (72) is likely to be a gross underestimate because $\rho\rho$, $\omega\omega$ and multibody channels were not included. Indeed, the absence of a signal in PS185 at CERN[46], suggests that two body final states constitute no more than $\sim 10\%$ of the total. In view of the uncertainties in the measured quantities this gives $br(\xi \to gg)$ consistent with unity for $J = 2$; for $J = 0$, it unacceptably exceeds the unitarity bound.

Thus we suggest that if evidence for the $\xi(2230)$ survives increases in statistics, the case $J = 2$ would be consistent with $\xi(2230)$ being a tensor glueball.

Such a result would have significant implications for the emergence of a glueball spectroscopy in accord with lattice QCD. It would also raise tantalising questions about the 0^{-+} sector, where lattice finds a glueball mass $\geq\ 2\ GeV$. This is interesting in view of the appearance of a clear 0^{-+} signal in the 1450 MeV region, which would then be difficult to reconcile with being the 0^{-+} glueball. We now turn to this question.

4.4 0^{-+} signals in $J/\psi \to \gamma R$

The branching ratio for $(Q\bar{Q})_V \to \gamma R$ in terms of the total gluonic decay width of the pseudoscalar state is given by eqs. 1 and 37

$$b_{rad}((Q\bar{Q})_V \to \gamma + R) = \Gamma_{R\to gg}\frac{x|H_{PS}(x)|^2}{8\pi(\pi^2-9)}\frac{m}{M^2}. \tag{73}$$

As noted in eq. (40), the above formula may be scaled as follows

$$10^3 br(J/\psi \to \gamma 0^{-+}) = (\frac{m}{1.5\ GeV})(\frac{\Gamma_{R\to gg}}{50\ MeV})\frac{x|H_{PS}(x)|^2}{45}. \tag{74}$$

This subsumes figs 2,3,5,6, 9 and 10 of ref.[18]. (Note that the dashed curve in fig 8 of ref. [18] corresponds to the above; the figure caption has typographical errors).

As a consistency test of this methodology for 0^{-+} states, we consider the production of the radial $q\bar{q}$ anticipated in this mass region[6]. The state $\eta(1295)$ ($\Gamma = 53\pm6$ MeV, with dominant decay into $\eta\pi\pi$) is a candidate on the grounds of mass and width[31]. The DM2 collaboration [47] may have evidence for the $\eta(1295)$ in their $J/\psi \to \gamma\eta\pi\pi$ data, which contains a peak in $\eta\pi\pi$ with the parameters $J^{PC} = 0^{-+}$, $m = 1265$, $\Gamma = 44\pm20MeV$, $br = (0.26\pm0.06)\times10^{-3}$. If this is the $\eta(1295)$, the scaled formula (40) then implies that $br(\eta(1295) \to gg) \sim 0.25$ if $\eta\pi\pi$ is the dominant decay mode. This is consistent with a $0^{-+}(q\bar{q})$ because $\eta\pi\pi$ dominance is expected[31].

[6]Note that the model cannot safely be applied to the η', whose total width is "accidentally" strongly suppressed because (apart from $\eta\pi\pi$, which is suppressed by three body phase space) only electromagnetic decays are non-negligible

At slightly higher mass, the $\eta(1440)$[5] is more prominently produced. Historically this was seen in $J/\psi \to \gamma K\bar{K}\pi$ with $br = (4.3 \pm 1.7) \times 10^{-3}$ [48]. The prominence of this state caused it to be identified as a potential glueball[7]. Subsequently it was realised that there are two states contained within this structure[5] whose individual production rates were smaller than the earlier, apparently large value. This development, together with improved lattice QCD estimates of the 0^{-+} glueball mass which place it above 2 GeV rather than in the 1.4 GeV region, caused the glueball interpretation to fall from favour. As noted in ref. [18], such a large production rate as originally reported[48] would seriously oversaturate $br(\eta(1440) \to gg)$, but subsequent separation of the signal into two resonances results in physically acceptable values for the individual gg widths.

The first analyses[49, 47] indicating the existence of additional structure in the $\eta(1440)$ region were, however, not in agreement. Recent data in $p\bar{p} \to \eta(1440) + \cdots$ help us to identify the problematic measurement and to propose a consistent picture that experiments should now pursue. We shall suggest that there are two states, η_L and η_H (for "Low" and "High" mass respectively), where η_L has significant coupling to glue while η_H is dominantly the $s\bar{s}$ member of the nonet, mixed with glue. Before giving the theoretical analysis, we survey the evidence from various experiments for $\eta_L(1410) \to \eta\pi\pi$ and $K\bar{K}\pi$, $\Gamma_{tot} \sim 50$ MeV and for $\eta_H(1480) \to K^*K$, $\Gamma_{tot} \sim 100$ MeV.

Obelix[13] sees two states in $p\bar{p} \to \pi\pi\eta_{L,H} \to \pi\pi(K\bar{K}\pi)$ with the properties

$$\eta_L(1416 \pm 2) \to K\bar{K}\pi; \Gamma_{tot} = 50 \pm 4 \ MeV \tag{75}$$

$$\eta_H(1460 \pm 10) \to K^*\bar{K}; \Gamma_{tot} = 105 \pm 15 \ MeV. \tag{76}$$

These values agree with the central values for the sighting by MarkIII[49] in J/ψ radiative decay. Combining errors in quadrature MarkIII finds

$$\eta_L(1416 \pm 10) \to a_0\pi \to K\bar{K}\pi; \Gamma_{tot} = 54^{+40}_{-30} \ MeV \tag{77}$$

$$\eta_H(1490 \pm 18) \to K^*\bar{K}; \Gamma_{tot} = 91 \pm 68 \ MeV. \tag{78}$$

Further evidence for the low mass state, in the decay channel $\eta\pi\pi$, comes from MarkIII[50] who find $\eta(1400 \pm 6)$, $\Gamma_{tot} = 47 \pm 13$ MeV; from DM2[47] who find

$\eta(1398 \pm 6)$, $\Gamma_{tot} = 53 \pm 11$ MeV; and the Crystal Barrel Collaboration[14]. The latter see $p\bar{p} \to \pi\eta(1410 \pm 3) \to \pi(\eta\pi\pi)$ with significant contribution in the glue favoured partial wave $\eta\sigma$[2]. Their value $\Gamma_{tot} = 86 \pm 10$ MeV is however substantially larger than the $\Gamma_{tot} \approx 50$ MeV found by the other experiments[13, 47, 50]. Possibly the differing production mechanisms of these experiments affects the apparent width of the resonance due to differing interference effects.

We now compute the production rate for these states in J/ψ radiative decay. For the η_L, MarkIII[49] see the decay mode $a_0\pi \to K\bar{K}\pi$ with

$$br(J/\psi \to \gamma\eta_L(1410) \to \gamma K\bar{K}\pi) = (0.66^{+0.29}_{-0.22}) \times 10^{-3}, \tag{79}$$

while a clear signal is found also in the $a_0\pi \to \eta\pi\pi$ channel by ref.[50]:

$$br(J/\psi \to \gamma\eta_L(1410) \to \gamma\eta\pi\pi) = (0.34 \pm 0.08) \times 10^{-3}. \tag{80}$$

These two channels are expected to dominate the decays. Adding them together and inserting into eq. (74) implies

$$\Gamma(\eta(1410) \to gg) = (54 \pm 13)\ MeV. \tag{81}$$

Combining the various width measurements, adding errors in quadrature, gives $\Gamma_{tot} = 54.2 \pm 3.4$ MeV, and hence

$$br(\eta(1410) \to gg) = 0.9 \pm 0.2. \tag{82}$$

For comparison, using just the larger width from Crystal Barrel[14] would give $br(\eta(1410) \to gg) = 0.65 \pm 0.2$. The data clearly indicate a strong coupling to gluons which argues against $\eta_L(1410)$ being pure $q\bar{q}$. Rather, it couples like a glueball, perhaps mixed with the nearby $q\bar{q}$ nonet.

Now we consider the $\eta_H(1480)$. This state decays into K^*K and is not seen in $\eta\pi\pi$.[7] Combining errors in quadrature, as above, MarkIII finds[49]

$$br((J/\psi \to \gamma\eta(1490) \to \gamma K^*K) = 1.03^{+0.33}_{-0.26}(10^{-3}) \tag{83}$$

[7]For this reason the DM2 analysis[47] indicating $\eta(1460) \to a_0\pi \to K\bar{K}\pi$ is suspect, since a genuine resonance decaying to $a_0\pi$ should also show up in $a_0\pi \to \eta\pi\pi$. Instead, in the $\eta\pi\pi$ channel only the $\eta(1410)$ is seen. Eliminating this DM2 state produces a harmonious picture given the remaining observations. We thank A.Kirk for discussions of this point.

whereby with $\Gamma_{tot} = 100 \pm 20$ MeV[13], eq. (74) implies

$$br(\eta(1490) \to gg) = (0.5 \pm 0.2)/br(K^* K). \tag{84}$$

Ref[31] anticipates that the radially excited $\eta^{s\bar{s}}$ should have a total width of up to 100 MeV, dominated by the channel K^*K. The above result, eq. (84), is compatible with such a state. Ref.[31] also finds that $\eta^{n\bar{n}}$ has suppressed width, decaying into $\eta\pi\pi$ through $a_0\pi$; this is compatible with the results on $\eta(1295)$ above. Thus a tentative interpretation of the pseudoscalar states is as follows:

$$\begin{aligned}
\eta(1295) \sim \eta^{n\bar{n}};\ br_{gg} \sim O(\alpha_s^2) \sim 0.25 \\
\eta_L(1410) \sim G(+q\bar{q});\ br_{gg} \sim 1 \\
\eta_H(1480) \sim \eta^{s\bar{s}}(+G);\ br_{gg} \sim 0.5
\end{aligned} \tag{85}$$

These conclusions can be sharpened if the widths and decays from Crystal Barrel and Obelix converge and if $J/\psi \to \gamma 0^{-+}$ is pursued further. We note also a new measurement from BEPC[35] which sees only a single state with a mass of 1467 ± 3 MeV, $\Gamma = 89 \pm 6$ MeV and $br(\psi \to \gamma\eta(1467) \to \gamma K\bar{K}\pi) = 1.86 \pm 0.10 \pm 0.4(10^{-3})$. Since $br(\eta(1467) \to K\bar{K}\pi) \leq 1$, this gives $br(\eta(1467) \to gg) \geq 1.1 \pm 0.2$. We urge that BEPC continue to investigate this state with a view to separating two signals: $\eta_H \to K^*K \to K\bar{K}\pi$ and $\eta_L \to a_0\pi \to K\bar{K}\pi$.

The experimental data on 0^{-+} production in radiative J/ψ decays in this mass region need clarification before strong conclusions can be drawn, but if the existence of two states in the $1400 - 1500$ MeV range, and their relative production (one or both much more strongly produced than $\eta(1295)$) is confirmed, we have a serious challenge to theoretical expectations. The experiments would appear to be telling us that the lightest pseudoscalar glueball is much lighter than predicted in quenched lattice QCD, (2.16 ± 0.27 GeV[43]). In view of the apparent possible success (within uncertainties noted above) of the lattice QCD predictions for the 0^{++} and 2^{++} glueball masses, such a discrepancy between lattice QCD and nature would be of great interest. We note that the mass and properties of the $\eta(1410)$ are consistent with predictions for a gluino-gluino bound state[17, 18], possibly mixed with nearby pseudoscalar $q\bar{q}$ states. If nature were supersymmetric and SUSY breaking did not violate R-invariance, the gluino mass would be $O(100)$ MeV[51, 52]. In that case the 0^{++} glueball would be in an approximate supermultiplet with the pseudoscalar gluino-gluino ($\tilde{g}\tilde{g}$) and spin-1/2 gluon-gluino bound states. This would lead to an "extra" isosinglet pseudoscalar in the spectrum, with mass around 1 1/2 GeV. Decay of such a $\tilde{g}\tilde{g}$ system would necessarily go through gluons, since its direct couplings to quarks would be suppressed by heavy squark masses and hadrons containing a single gluino would be too massive to be pair produced by the $\tilde{g}\tilde{g}$. Thus $br(\tilde{g}\tilde{g} \to gg) \sim 1$.

Improving the data on these states would provide important constraints. It is now a clear challenge for experiment to separate and quantify these signals.

5 Constraints on Glueballs from $\gamma\gamma$ Widths

5.1 $R \to \gamma\gamma$ and $J/\psi \to \gamma R$

If a state R_J is a glueball (or light gluinoball), it will occur in $\psi \to \gamma R_J$ as a singleton and be strongly suppressed in $R_J \to \gamma\gamma$. By contrast, if R_J is an $I = 0$ member of a $q\bar{q}$ nonet there will be two orthogonal states in the singlet - octet flavour basis available for production both in $J/\psi \to \gamma R_J$ and $R_J \to \gamma\gamma$. Flavour $1 - 8$ mixing angles may suppress one or the other of the pair in either $R_J \to \gamma\gamma$ or in $J/\psi \to \gamma R_J$ but there are strong correlations between the two processes so that a comparison of the two processes can help to distinguish glueball from $q\bar{q}$. In particular, if a $q\bar{q}$ state is flavour "favoured" in $J/\psi \to \gamma q\bar{q}$, so that it is prominent and superficially somewhat "glueball - like", it will also be flavour favoured in $\gamma\gamma \to R(q\bar{q})$ (see below) in dramatic contrast to a glueball.

For heavy $Q\bar{Q}$ resonances such as the χ_c states, the amplitudes $< gg|R >$, which enter the computation of $b_{rad}(Q\bar{Q}_V \to \gamma + R)$, and $< R|\gamma\gamma >$, which in principle can be measured as a function of photon virtualities, are identical except for the value of the overall coefficient A_{JPC}. The relative rates (see eq. (36 and sec. 2.4) would be (in leading order)

$$\Gamma(R \to \gamma\gamma) = \frac{9e_Q^4}{2}(\frac{\alpha}{\alpha_s})^2\Gamma(R \to gg)$$

where e_Q is the relevant quark charge. Bearing in mind the limitations to use of this relation for light $q\bar{q}$ mesons discussed in sec. 2.4, (see also ref.[53]) we shall tentatively adopt this relation and test it against the known $f_2(1270; 1525)$. Finding it to be qualitatively reasonable, we apply it in section 6 to $f_0(1370, 1500, 1710)$, allowing for mixing between $n\bar{n}$, $s\bar{s}$, and gg.

5.2 Orthogonal $q\bar{q}$ mesons coupling to $\gamma\gamma$ and gg

Define

$$R_I \equiv cos\theta|1\rangle + sin\theta|8\rangle \tag{86}$$

$$R_{II} \equiv cos\theta|8\rangle - sin\theta|1\rangle, \tag{87}$$

where $|1, 8\rangle$ denote the $SU(3)$ flavor $q\bar{q}$ states. (This is a more natural basis for what follows than the ideal flavour basis used in section 3.2). Then in terms of the intrinsic rates for a single $q\bar{q}$ flavour ($u\bar{u}; d\bar{d}$ or $s\bar{s}$ assumed to be of equal strength)[8]

$$\Gamma(R_I \to gg) = \Gamma(q\bar{q} \to gg) \times 3\ cos^2\theta \tag{88}$$

and

$$\Gamma(R_{II} \to gg) = \Gamma(q\bar{q} \to gg) \times 3\ sin^2\theta, \tag{89}$$

while the $\gamma\gamma$ widths are in a different proportion. Defining $\Gamma(q\bar{q} \to \gamma\gamma)$ to be the $\gamma\gamma$ width for quarks of unit electric charge:

$$\Gamma(R_I \to \gamma\gamma) = \Gamma(q\bar{q} \to \gamma\gamma)(cos\theta\frac{2}{3\sqrt{3}} + sin\theta\frac{1}{3\sqrt{6}})^2 \tag{90}$$

[8]The factor 3 in this equation reflects the $1/\sqrt{3}$ projection of each of the three $q\bar{q}$ flavors in the flavor singlet state.

and

$$\Gamma(R_{II} \to \gamma\gamma) = \Gamma(q\bar{q} \to \gamma\gamma)(-sin\theta\frac{2}{3\sqrt{3}} + cos\theta\frac{1}{3\sqrt{6}})^2. \tag{91}$$

In a form that shows the relation to the gg widths,

$$\Gamma(R_I \to \gamma\gamma) = \Gamma(q\bar{q} \to \gamma\gamma) \times \frac{1}{6}cos^2(\theta - \tau) \tag{92}$$

and

$$\Gamma(R_{II} \to \gamma\gamma) = \Gamma(q\bar{q} \to \gamma\gamma) \times \frac{1}{6}sin^2(\theta - \tau), \tag{93}$$

where $\tau \equiv tan^{-1}\frac{1}{2\sqrt{2}} \sim 19.5^o$.

It is clearly possible for an individual $q\bar{q}$ to decouple "accidentally" in gg or $\gamma\gamma$ if $\theta \sim 0$ or 19.5^o. However for the orthogonal ***system*** we have the sum rule

$$\Gamma(R_I \to \gamma\gamma) + \Gamma(R_{II} \to \gamma\gamma) = \frac{1}{6}\Gamma(q\bar{q} \to \gamma\gamma) \tag{94}$$

and

$$\Gamma(R_I \to gg) + \Gamma(R_{II} \to gg) = 3\Gamma(q\bar{q} \to gg). \tag{95}$$

Thus using eq. 36 and including the next order QCD corrections,

$$\frac{\Gamma(R_I \to \gamma\gamma) + \Gamma(R_{II} \to \gamma\gamma)}{\Gamma(R_I \to gg) + \Gamma(R_{II} \to gg)} = \frac{\alpha^2}{4\alpha_s^2} \times (1 + c\frac{\alpha_s}{\pi})^{-1} \tag{96}$$

where $c \sim -0.4$ for tensors and ~ 8.6 for scalars[29, 54].

In the case of tensors the input data are

$$\Gamma(f_2(1270) + f_2(1525)) \to \gamma\gamma) = 3.0 \pm 0.4\ keV \tag{97}$$

and, from our analysis in section 3.2,

$$\Gamma(f_2(1270) + f_2(1525)) \to gg) = 58 \pm 8\ MeV. \tag{98}$$

With these widths, eq. 96 gives $\alpha_s^{eff} \sim 0.48 \pm 0.05$, not unreasonable for this mass region[55].

Considering the ratio of the $\gamma\gamma$ and gg widths of the entire orthogonal system allowed us to extract α_s^{eff}, with little sensitivity to θ. We can instead employ ratios of the R_I and R_{II} $\gamma\gamma$ and gg widths to extract θ with little sensitivity to α_s^{eff}. For an orthogonal $q\bar{q}$ pair, eqs. 90-93 imply

$$\frac{\Gamma(R_I \to gg)}{\Gamma(R_{II} \to gg)} = \frac{1}{tan^2\theta} \tag{99}$$

while

$$\frac{\Gamma(R_I \to \gamma\gamma)}{\Gamma(R_{II} \to \gamma\gamma)} = \frac{1}{tan^2(\theta - 19.5^o)}. \tag{100}$$

Note that the two sets of equations will not give the same value of θ if our procedure is not valid. As a consistency check, we determine θ both ways for the f_2 states. The $J/\psi \to \gamma R$ data gave us

$$\frac{\Gamma(f_2(1270) \to gg)}{\Gamma(f_2(1525) \to gg)} = \frac{41 \pm 7MeV}{17 \pm 2MeV} \to \theta = (33 \pm 2)^o, \tag{101}$$

while $\gamma\gamma$ data give

$$\frac{\Gamma(f_2(1270) \to \gamma\gamma)}{\Gamma(f_2(1525) \to \gamma\gamma)} = (26.2 \pm 2.8) \pm 26\% \to \theta = (30.5 \pm 2)^o. \tag{102}$$

The consistency of these results encourages us to apply the ideas to scalar mesons. However, the presence of possibly three scalar states in close proximity, $f_0(1370; 1500)$ and $f_{J=0?}(1710)$, and in the vicinity of the lattice scalar glueball, suggests that mixing involving both $q\bar{q}$ and gg will be essential. We shall now consider this situation.

5.3 $q\bar{q}$ nonet and glueball coupling to $\gamma\gamma$ and gg

If the $q\bar{q}$ nonet, $R_{I,II}$ is in the vicinty of a glueball, G, the above analysis requires generalisation. Three isoscalars arise. With $R_{I,II}$ as above, the mixed states may be written

$$\begin{aligned}
\Psi_3 &= cos\beta|R_{II}\rangle - sin\beta|G\rangle \\
\Psi_2 &= cos\gamma|R_I\rangle - sin\gamma(cos\beta|G\rangle + sin\beta|R_{II}\rangle) \\
\Psi_1 &= sin\gamma|R_I\rangle + cos\gamma(cos\beta|G\rangle + sin\beta|R_{II}\rangle)
\end{aligned} \tag{103}$$

If we ignore mass and phase space effects, and any differences between the $n\bar{n}$ and $s\bar{s}$ wavefunctions, then proceeding as in the previous section we obtain

$$\begin{aligned}\Gamma((\Psi_1+\Psi_2)\to\gamma\gamma) &= \Gamma(q\bar{q}\to\gamma\gamma)\times(\frac{1}{6}cos^2(\theta-\tau)+\frac{1}{6}sin^2\beta sin^2(\theta-\tau)) \\ \Gamma(\Psi_3\to\gamma\gamma) &= \Gamma(q\bar{q}\to\gamma\gamma)\times\frac{1}{6}sin^2(\theta-\tau)cos^2\beta \end{aligned} \tag{104}$$

Defining $\Gamma_{\gamma\gamma}\equiv\Sigma_{i=1}^3\Gamma(\Psi_i\to\gamma\gamma)$ and later Γ_{gg} analogously, the generalisation of eq.(94) becomes

$$\Gamma_{\gamma\gamma}=\frac{1}{6}\Gamma(q\bar{q}\to\gamma\gamma). \tag{105}$$

The generalisation of the relation for the gluon couplings, eq(95), becomes

$$\Gamma_{gg}=3\Gamma(q\bar{q}\to gg)+\Gamma(G\to gg). \tag{106}$$

Consequently

$$\Gamma_{\gamma\gamma}=\frac{\alpha^2}{4\alpha_s^2}(1+c\frac{\alpha_s}{\pi})^{-1}(\Gamma_{gg}-\Gamma(G\to gg)). \tag{107}$$

Thus, specialising to 0^{++} mesons, the experimentally measurable quantities $\Gamma_{\gamma\gamma}$ and Γ_{gg} obey the relation

$$\Gamma_{\gamma\gamma}[keV]\leq\frac{(0.5/\alpha_s)^2}{20(1+8.6\frac{\alpha_s}{\pi})}\Gamma_{gg}[\ MeV]. \tag{108}$$

A major uncertainty comes from the large higher order QCD correction for the 0^{++} sector which reduces the right hand side by a factor of approximately 2.2 To be conservative we therefore work to leading order. If $f_0(1370)$ is one of the trinity of glue associated states, then we infer from $\Gamma(f_0\to\gamma\gamma)=5.4\pm2.3$ keV[5] that

$$\Gamma_{gg}\geq108\pm46\ MeV \tag{109}$$

or from eq(38), neglecting mass dependence:

$$br(J/\psi\to\gamma+\Sigma_{i=1}^3f_0^i)\geq(1.1\pm0.5)10^{-3}. \tag{110}$$

Since $br(J/\psi\to\gamma f_J(1710))\to K\bar{K}=0.97\pm0.12\ 10^{-3}$[5], this bound is satisfied by present data if the 1710 has $J=0$, even if $br(f_0(1710)\to K\bar{K})\sim1$

and there is negligible production of $f_0(1500)$ in radiative decay. However the limit is only barely respected so unless $\Gamma(f_0(1500) \to \gamma\gamma) + \Gamma(f_0(1710) \to \gamma\gamma)$ is very small, either $br(J/\psi\gamma f_0(1500))$ and or $br(J/\psi\gamma f_0(1370))$ must be non-negligible, or $br(f_0(1710) \to K\bar{K}) < 1$. If the 1710 state proves to have $J = 2$, the bound (110) will be very stringent indeed. We now consider specific examples of mixing in the $f_0(1370; 1500; 1710)$ system.

6 Three-State Mixings

An interesting possibility is that three f_0's in the $1.4 - 1.7$ GeV region are admixtures of the three isosinglet states gg, $s\bar{s}$, and $n\bar{n}$[2]. Recently there have been two specific schemes proposed which are based on lattice QCD and the emergent phenomenology of scalar mesons. In this section we present a simplified formalism for treating a three component system of this type.

At leading order in the glueball-$q\bar{q}$ mixing, ref[2] obtained

$$\begin{aligned} N_G|G\rangle &= |G_0\rangle + \xi(\sqrt{2}|n\bar{n}\rangle + \omega|s\bar{s}\rangle) \\ N_s|\Psi_s\rangle &= |s\bar{s}\rangle - \xi\omega|G_0\rangle \\ N_n|\Psi_n\rangle &= |n\bar{n}\rangle - \xi\sqrt{2}|G_0\rangle \end{aligned} \tag{111}$$

where the N_i are appropriate normalisation factors, $\omega \equiv \frac{E(G_0)-E(d\bar{d})}{E(G_0)-E(s\bar{s})}$ and the mixing parameter $\xi \equiv \frac{\langle d\bar{d}|V|G_0\rangle}{E(G_0)-E(d\bar{d})}$. Our analysis suggests that the $gg \to q\bar{q}$ mixing amplitude manifested in $\psi \to \gamma R(q\bar{q})$ is $O(\alpha_s)$, so that qualitatively $\xi \sim O(\alpha_s) \sim 0.5$. Such a magnitude implies significant mixing in eq.(111) and is better generalised to a 3×3 mixing matrix. Mixing based on lattice glueball masses lead to two classes of solution of immediate interest:

(i)$\omega \leq 0$, corresponding to G_0 in the midst of the nonet[2]

(ii)$\omega > 1$, corresponding to G_0 above the $q\bar{q}$ members of the nonet[45].

We shall denote the three mass eigenstates by R_i with $R_1 = f_0(1370)$, $R_2 = f_0(1500)$ and $R_3 = f_0(1710)$, and the three isosinglet states ϕ_i with

$\phi_1 = n\bar{n}$, $\phi_2 = s\bar{s}$ and $\phi_3 = gg$ so that $R_i = f_{ij}\phi_i$. Recent data on the decay $f_0(1500) \to K\bar{K}$[38] may be interpreted within the scheme of ref[2] as being consistent with the parameter $\omega \sim -2$. This enables simple analysis; if for illustration we adopt $\xi = 0.5 \sim \alpha_s$, the resulting mixing amplitudes are (scheme "A"):

	f_{i1}	f_{i2}	f_{i3}
$f_0(1370)$	0.86	0.13	-0.50
$f_0(1500)$	0.43	-0.61	0.61
$f_0(1710)$	0.22	0.76	0.60

By contrast, Weingarten[45] has considered the case where the bare glueball lies above the $s\bar{s}$ member of the nonet. His mixing matrix is (scheme "B"):

	f_{i1}	f_{i2}	f_{i3}
$f_0(1370)$	0.87	0.25	-0.43
$f_0(1500)$	-0.36	0.91	-0.22
$f_0(1710)$	0.34	0.33	0.88

The solutions for the lowest state are similar, as are the relative phases and qualitative importance of the G component in the high mass state. Both solutions exhibit destructive interference between the $n\bar{n}$ and $s\bar{s}$ flavours for the middle state.

If we make the simplifying assumption that the photons couple to the $n\bar{n}$ and $s\bar{s}$ in direct proportion to the respective e_i^2 (i.e. we ignore mass effects and any differences between the $n\bar{n}$ and $s\bar{s}$ wavefunctions), then the corresponding two photon widths can be written in terms of these mixing coefficients:

$$\Gamma(R_i) = |f_{i1}\frac{5}{9\sqrt{2}} + f_{i2}\frac{1}{9}|^2\Gamma, \tag{112}$$

where Γ is the $\gamma\gamma$ width for a $q\bar{q}$ system with $e_q = 1$. One can use eq. (112) to evaluate the relative strength of the two photon widths for the three f_0 states with the input of the mixing coefficients. These are (ignoring mass dependent effects)

$$f_0(1370) : f_0(1500) : f_0(1710) \sim 12 : 1 : 3 \tag{113}$$

in scheme A, to be compared with

$$f_0(1370) : f_0(1500) : f_0(1710) \sim 13 : 0.2 : 3 \tag{114}$$

in scheme B. At present the only measured $\gamma\gamma$ width in this list is that of the $f_0(1370) = 5.4 \pm 2.3$ keV[5]. Using this to normalise the above, we anticipate $f_0(1500) \to \gamma\gamma \sim 0.5$ keV (scheme A) or ~ 0.1 keV (scheme B). Both schemes imply $\Gamma(f_0(1710) \to \gamma\gamma) = 1 - 2$ keV.

This relative ordering of $\gamma\gamma$ widths is a common feature of mixings for all initial configurations for which the bare glueball does not lie nearly degenerate to the $n\bar{n}$ state. As such, it is a robust test of the general idea of $n\bar{n}$ and $s\bar{s}$ mixing with a lattice motivated glueball. If, say, the $\gamma\gamma$ width of the $f_0(1710)$ were to be smaller than the $f_0(1500)$, or comparable to or greater than the $f_0(1370)$, then the general hypothesis of significant three state mixing with a lattice glueball would be disproven. The corollary is that qualitative agreement may be used to begin isolating in detail the mixing pattern.

Now we turn to J/ψ radiative decay rates. Since in either scheme

$$\Gamma_{\gamma\gamma} = 7.5 \pm 2.8\ keV, \tag{115}$$

the discussion of the previous section implies,

$$br(J/\psi \to \gamma \Sigma f_0) \geq (1.5 \pm 0.6) \times 10^{-3}. \tag{116}$$

However each scheme makes a more specific prediction. By our hypothesis that $q\bar{q}$ coupling to gg is suppressed at $O(\alpha_s)$ relative to the corresponding glueball amplitude, we may scale the $J/\psi \to \gamma f_0$ production amplitudes for the mixed states as follows. For simplicity we shall assume that $A(gg \to n\bar{n}) = \sqrt{2}A(gg \to s\bar{s}) = c\alpha_s A(gg \to G)$, where c is some constant whose magnitude and phase are in general model dependent. In this approximation, we have for scheme A

$$\begin{aligned} A(f_0(1370) \to gg) &= (-0.5 + c\alpha_s 1.3)A_0 \\ A(f_0(1500) \to gg) &= 0.6A_0 \\ A(f_0(1710) \to gg) &= (0.6 + c\alpha_s 1.1)A_0 \end{aligned}$$

In general we see that for mixing scheme A:

(i)The absence of a dominant signal in $J/\psi \to \gamma f_0(1370)$ suggests that c is not negative and that the G-$q\bar{q}$ interference there is destructive.

(ii)The $q\bar{q}$ admixture in the $f_0(1500)$ is nearly pure flavour octet and hence decouples from gg. This leaves the strength of $br(J/\psi \rightarrow \gamma f_0(1500))$ at about 40% of the pure glueball strength, which is consistent with the mean of the two analyses in section 4.1.

(iii)The destructive interference in the $f_0(1370)$ case implies a constructive effect for the $f_0(1710)$ and hence this picture predicts that $br(J/\psi \rightarrow \gamma f_0(1710)) > br(J/\psi \rightarrow \gamma f_0(1500)) > br(J/\psi \rightarrow \gamma f_0(1370))$. If as a particular example for comparison between the two schemes we take $c\alpha_s = 0.5/1.3$ to decouple $f_0(1370)$ entirely in radiative J/ψ decay, we find $br(J/\psi \rightarrow \gamma f_0(1710)) : br(J/\psi \rightarrow \gamma f_0(1500) : br(J/\psi \rightarrow \gamma f_0(1370)) = 1.1 : 0.4 : 0$.

Mixing scheme B, corresponding to an ideal glueball lying above the nonet, leads to the following amplitudes:

$$\begin{aligned}
A(f_0(1370) \rightarrow gg) &= (-0.4 + c\alpha_s 1.5)A_0 \\
A(f_0(1500) \rightarrow gg) &= (-0.2 + c\alpha_s 0.4)A_0 \\
A(f_0(1710) \rightarrow gg) &= (0.9 + c\alpha_s 0.8)A_0
\end{aligned}$$

Here both $f_0(1370)$ and $f_0(1500)$ production are suppressed due to the destructive interference of the glueball and $q\bar{q}$ components; the $f_0(1710)$ being enhanced as in the previous example. For the example $c\alpha_s = 0.4/1.5$, (chosen to decouple the $f_0(1370)$ and enable comparison with scheme A as above) we find $br(J/\psi \rightarrow \gamma f_0(1710)) : br(J/\psi \rightarrow \gamma f_0(1500) : br(J/\psi \rightarrow \gamma f_0(1370)) = 1.2 : 0.01 : 0$.

Thus, in conclusion, both these mixing schemes imply a similar hierachy of strengths in $\gamma\gamma$ production which may be used as a test of the general idea of three state mixing between glueball and a nearby nonet. Prominent production of $J/\psi \rightarrow \gamma f_0(1710)$ is also a common feature. When the experimental situation clarifies on the $J/\psi \rightarrow \gamma f_0$ branching fractions, we can use the relative strengths to distinguish between the case where the glueball lies within a nonet, ref[2], or above the $s\bar{s}$ member, ref[45].

7 Summary

We have clarified the relationship between $b_{rad}(Q\bar{Q}_V \rightarrow \gamma + R)$ and $br(R \rightarrow gg)$ proposed by Cakir and Farrar[18]. In particular, we have examined its dependence on the $< gg|R >$ form factors and discussed theoretical and experimental constraints on these form factors. We conclude that the relation can be used, possibly with generalized $H_J(x)$ functions, for light-$q\bar{q}$ mesons and glueballs as well as heavy $q\bar{q}$ mesons. Using this relation, we find

- The $f_0(1500)$ is at least half-glueball if the Bugg et al analysis[3] of the 4π channel is confirmed, but is less so according to the BES results. Analysis of MarkIII data on $J/\psi \rightarrow \gamma\pi\pi$ is urgently needed. At this moment the experimental determinations of $\Gamma(J/\psi \rightarrow \gamma f_0(1500))$ are inconsistent.

- The $f_J(1710)$ is also at least half-glueball, if $J = 0$; if $J = 2$ it is a $q\bar{q}$ meson. Experimental determinations of the $f_{0,2}$ spectra in the $1.6 - 1.8$ GeV region are presently inconsistent.

- The $\xi(2330)$ is unlikely to have $J = 0$, if present experimental data are correct. If it has $J = 2$ it strongly resembles a glueball.

- The $\eta(1440)$ is separated into two states. The lower mass state, $\eta_L(1410)$, has strong affinity for glue; the higher mass $\eta_H(1480)$ is consistent with being the $s\bar{s}$ member of a nonet, perhaps mixed with glue.

It is of urgent importance to (a) arrive at an experimental consensus on the f_0 and f_2 masses and widths in the 1600-1800 region and (b) resolve the discrepancies in the present determinations of $br(\psi \rightarrow \gamma f_0(1500))$. Measurement of production branching fractions of the f_0 and f_2 mesons in Υ radiative decay should be quite easy and yield useful additional information. We also outlined a procedure to use data on $\psi \rightarrow \gamma R$ and $\gamma\gamma \rightarrow R$ together, to help unravel the $q\bar{q}$ and gg composition of mesons. To accomplish this, measurement of $\Gamma(f_0(1370; 1500; 1710) \rightarrow \gamma\gamma)$ is an essential ingredient.

An emerging mystery is the $\eta(1440)$ region. Its properties seem to differ in J/ψ radiative decay and $p\bar{p}$ annihilation, and it has not been seen in central production. Possibly these differences are due to the different interplay of gluon and $q\bar{q}$ annihilation in the various production processes. This merits further investigation, both experimental and theoretical. The strong production of the $\eta(1410)$ in radiative J/ψ decay indicate that it could be a glueball. However its low mass is difficult to reconcile with lattice gauge predictions. Its properties and mass are consistent with those expected for a bound state of light gluinos. Given that the $\eta(1410)$ may be evidence of a new degree of freedom in QCD, or evidence of dynamics beyond quenched lattice gauge theory in the 0^{-+} sector, more detailed experimental investigation of the pseudoscalar sector is a high priority.

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Appendix: Analytical Expressions for $H_J(x)$

The analytical expressions for the loop integral $H_J(x)$ are given in Ref. [19]. In the normalisation of the present paper they are:

$$H_{0^{-+}}(x) = \frac{4}{x}\left[L(1-2x) - L(1) - \frac{1-x}{2-x}(2L(1-x) - \frac{\pi^2}{3} + \frac{1}{2}\ln^2(1-x)) - \frac{x}{1-2x}\ln(2x)\right] + i4\pi\frac{1-x}{(2-x)x}\ln(1-x) \tag{117}$$

for $J = 0^{-+}$, and

$$H_{0^{++}} = \sqrt{\frac{2}{3}}\left[\frac{2-3x}{x^2} + \left(10\frac{1-x}{x^3} + 4\frac{1-2x}{x^2}\ln(2)\right)\ln(1-x)\right.$$

$$
+\left(\frac{8}{x^2}+2\frac{1-x}{x(1-2x)}\right)\ln(2x).-3\frac{1-x}{x(2-x)}\ln^2(1-x)
$$
$$
+\frac{8-6x+x^2-6x^3}{x^3(2-x)}\pi^2/6-\frac{4-5x+2x^2}{x^3}L(1-2x)
$$
$$
-4\frac{2-2x-x^2}{x^2(2-x)}L(1-x)+i\pi 6\frac{1-x}{x(2-x)}\ln(1-x)\Bigg] \tag{118}
$$

for 0^{++}, where $L(x)$ is a Spence function, defined as

$$
L(x)=-\int_0^x\frac{dx}{x}\ln(1-x). \tag{119}
$$

There are three helicity amplitudes for the tensor state, and they are related to the total $H_{2^{++}}$ by

$$
|H_{2^{++}}(x)|^2=|H^0_{2^{++}}(x)|^2+|H^1_{2^{++}}(x)|^2+|H^2_{2^{++}}(x)|^2. \tag{120}
$$

The helicity amplitudes $H^l_{2^{++}}$ in Eq. 120 are

$$
\begin{aligned}
H^0_{2^{++}} = \frac{2\sqrt{3}}{x^3}\Bigg[& x(6-5x)+\frac{2}{3}\frac{6-19x+18x^2}{x}(1-x)\ln(1-x) \\
& -\frac{10-12x+5x^2}{3(2-x)}g_1+\frac{2}{3}\frac{6-38x+71x^2-37x^3}{1-2x}\ln(2x) \\
& -8\frac{(1-x)^2}{x^2(2-x)}g_2+\frac{4}{3}\frac{6-6x-x^2}{x}\left(\ln(2)-\frac{1}{2}i\pi\right) \\
& -\frac{4}{3}(12-26x+13x^2)g_3\Bigg],
\end{aligned} \tag{121}
$$

$$
\begin{aligned}
H^1_{2^{++}} = \frac{2\sqrt{1-x}}{x^3}\Bigg[& -\frac{1}{3}(38-9x)x-\frac{2}{x}(4-13x+16x^2-4x^3)\ln(1-x) \\
& -2\frac{x(1-x)}{2-x}g_1-\frac{4}{1-2x}(2-11x+16x^2-4x^3)\ln(2x) \\
& +8\frac{(1-x)(2-2x+x^2)}{x^2(2-x)}g_2-\frac{16}{3}\frac{3-3x+x^2}{x}\left(\ln(2)-\frac{1}{2}i\pi\right) \\
& +4(8-12x+3x^2)g_3\Bigg]
\end{aligned} \tag{122}
$$

and

$$H^2_{2^{++}} = \frac{\sqrt{2}(1-x)}{x^3}\left[\frac{16}{3}x + \frac{4}{x}(1-6x+6x^2)\ln(1-x) + 2\frac{5-6x+2x^2}{2-x}g_1 \right.$$
$$+4(1-6x)\ln(2x) - 4\frac{2-4x+6x^2-4x^3+x^4}{x^2(2-x)}g_2$$
$$\left. +\frac{4}{3}\frac{6-6x+11x^2}{x}\left(\ln(2)-\frac{1}{2}i\pi\right) - 16(1-x)g_3\right] \quad (123)$$

where

$$g_1 = L(1) - L(1-2x), \quad (124)$$

$$g_2 = L(1-2x) - 2L(1-x) + L(1) - \frac{1}{2}\ln^2(1-x) + i\pi\ln(1-x) \quad (125)$$

and

$$g_3 = L(1-x) - L(1-2x) - \ln(2)\ln(1-x) \quad (126)$$

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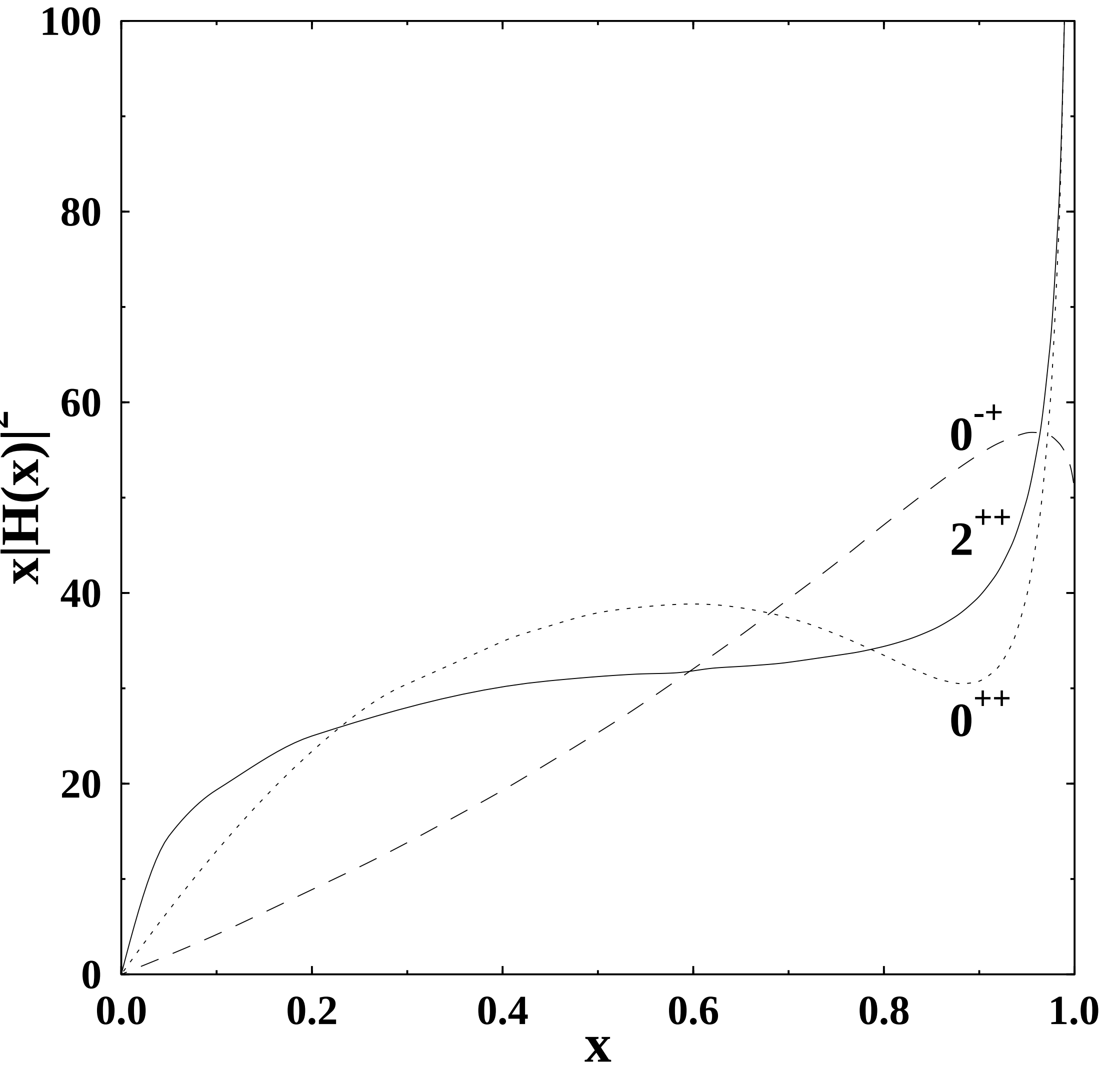

Figure 1: Magnitude of the loop integral, $x|H|^2$ versus x for 0^{++} (dotted), 0^{-+} (dashed) and 2^{++} (solid); $x = 1 - (\frac{m_R}{m_V})^2$.