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The production and decay of hybrid mesons by flux-tube breaking

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Abstract

An analytic calculation of the breaking of excited chromoelectric flux-tubes is performed in an harmonic oscillator approximation and applied to predict the dynamics of all J^{PC} low-lying gluonic excitations of mesons (hybrids). Widths, branching ratios and production dynamics of some recently discovered $J^{PC} = 1^{-+}, 0^{-+}$ and 1^{--} mesons are found to be in remarkable agreement with these results. We introduce the selection rules that can be used to understand the systematics of numerical decay calculations and we find possible significant breaking of these rules for specific channels that may enable enhanced production and detection of hybrids.

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1 Introduction

Alongside justifiable pride in establishing and applying the standard model, we should also recognise that there remains an area of substantial fundamental ignorance: while the gluon degrees of freedom expressed in L_{QCD} have been established beyond doubt in high momentum data, their dynamics in the strongly interacting limit epitomised by hadron spectroscopy are quite obscure. It is possible that this is about to change as candidates for gluonic hadrons (glueballs and hybrids) are now emerging [1].

For the first time there is a candidate scalar glueball [2, 3] whose mass $1.5 \sim 1.6$ GeV is consistent with the prediction of 1550 ± 50 MeV from lattice studies of QCD [4]. Simulations of the lattice dynamics, where gluonic fields are modelled as flux-tubes, reproduce these numbers for glueballs [5] and predict that hybrid mesons will be manifested in the $1.5 - 2$ GeV mass range [5, 6]. This is where candidates are now emerging, in particular the predicted [7, 8, 5] family of lowest lying multiplets of hybrid hadrons including $J^{PC} = (0,1,2)^{-+}, 1^{--}$. Of these states, potentially the most clear cut as a hybrid would be the $J^{PC} = 1^{-+}$ exotic in data from the AGS at Brookhaven with mass of about 2 GeV [9]. Less unique signatures but nonetheless potential 0^{-+} and 2^{-+} partners are seen in this same mass region in diffractive production by the VES Collaboration at Serpukhov [10, 11]. Extensive and thorough analyses of the 1^{--} system, which is especially well probed experimentally due to its isolation in e^+e^- annihilation and photoproduction, show that in the 1.4 to 2 GeV range of interest "mixing with non-$q\bar{q}$ states must occur" [12]. If lattice simulations [13] and the modelling of hybrid decays are reliable, the production and decay of charmonium hybrids at the Tevatron may be responsible, in part, for the anomalous production of ψ and ψ' observed at CDF [14].

Predictions for masses and/or the J^{PC} patterns of multiplets are a guide to identifying potential candidates for hybrids and glueballs but alone will not establish the spectroscopy and dynamics. Characteristic production and decay signatures proved seminal in establishing the light quark $\bar{Q}Q$ nonets and will be no less important in exciting and recognising gluonic hadrons. While the masses of glueballs and hybrids are computable in lattice QCD, at least in the quenched approximation, the decay dynamics are at present beyond its reach. However, intuition gained from the strong coupling expansion of lattice QCD has inspired the development of flux-tube models of mesons, which are probably the nearest we have to a realistic simulation of strong gluon dynamics, whereby the decay amplitudes may be computed [15]. In this picture mesons consist of $Q\bar{Q}$ connected by a cylindrical bag of coloured fields: the "flux-tube". When the flux tube is in its ground state, the excitation of the $Q\bar{Q}$ degree of freedom yields the conventional meson spectrum; excited modes of the flux-tube are also natural in strong QCD and this leads to a set of states that have yet to be confirmed by experiment. It is the existence of these "hybrid" states (where the flux-tube is excited or "plucked" in the presence of the $Q\bar{Q}$ coloured sources) that remains an open question within QCD dynamics and, as such, a missing part of the standard model.

Isgur, Paton and collaborators have developed and applied this model with some success to the decays of conventional mesons [15] and have also given some limited predictions for the decays of a few hybrid states [16], specifically those with light flavours and exotic J^{PC} quantum numbers. It is the latter that have in part motivated the search strategy for hybrids [9, 17]. This was adopted by the BNL experiment [9] who have studied the production of $\pi f_1(1285)$ and find that over 40% of the signal is in the exotic 1^{-+} partial wave; however the production mechanism appears to involve ρ exchange and is in sharp contrast to the expectations.

It is therefore timely to examine in more detail the implications of the model and the experimental signatures.

A powerful and empirically successful approach [15, 18], has been to use S.H.O. basis wavefunctions, thereby enabling analytic studies that reveal the relationships among amplitudes. This has been employed for the flux-tube model in the case of the decays of conventional mesons [15] and the point of departure for the present paper will be to make an analogous application to the dynamics of hybrid mesons. We find that this analytic approximation

1. reproduces the numerical results of Isgur, Kokoski and Paton [16] with rather good accuracy,
2. reveals for the first time the relationships that exist among amplitudes and which underpin their relative magnitudes, thereby highlighting signals that are potentially significant on general grounds rather than due to specific choices of parameters
3. exhibits the explicit dependence of amplitudes and widths on masses and other parameters thereby enabling application to possible candidates as they emerge, in particular the $(0,1,2)^{-+}$ states recently reported from VES and BNL [10, 9] and is immediately extendable to heavy quark hybrids
4. provides the first detailed analysis of hybrid decays for both light and heavy flavours, for arbitrary relevant quantum numbers.

We find that the possibility of the VES 0^{-+} being a hybrid [11] is strongly supported by our analysis whereas the VES 2^{-+} state is less clear. The 1^{--} partial wave shows clear signals consistent with hybrid excitation in the branching ratios of $\rho(1460)$ [12]. Furthermore we find that the selection rule that hybrids do not decay into ground state mesons may in some cases (notably $\pi\rho$ decays) be significantly broken. This may explain the production mechanism of the 1^{-+} candidate at BNL [9] and it also suggests new ways of accessing the hybrid sector.

Having obtained analytic closed forms for the decay amplitudes it is possible to apply these results to the dynamics of heavy hybrids, in particular hybrid charmonium. Heavy hybrids are specifically interesting, because heavy quarkonium is well understood, and the excitations of gluonic degrees of freedom for charmonium are predicted to be manifested in the vicinity of $DD \sim DD^{**}$ charm thresholds [6] and so should be clean experimentally. We shall report on this elsewhere [19] as the numerical results depend in part on parameters that need first to be confronted with the dynamics of the conventional charmonia for which there is no analogous discussion in the literature known to us [20]. In the present paper we shall restrict our attention primarily to the light flavoured hybrids, exposing the analytic and parameter dependence of the decay amplitudes, and both updating and extending the numerical studies of ref. [16] in the light of modern data. As already mentioned, this will expose the dynamical origin of the familiar selection rules for hybrid decays and, more important, reveal significant violations of them for certain decays or production channels.

The structure of the paper is as follows.

In section 2 we present the general formalism describing mesons in flux-tube models, for both conventional and hybrid configurations. This leads to the master equation (eqn. 3) controlling the structure of decay amplitudes. Section 3 shows how this leads to selection rules, in particular that the dominant two-body decays of the lowest hybrids are into $L = 0$

and $L = 1$ ("S + P") $q\bar{q}$ meson pairs. These dominant channels are studied in some detail in section 4 where we extract the analytic structures, reproduce existing computations for exotic J^{PC} hybrids and extend them to all low-lying $J^{PC} (J \leq 2)$ combinations for both light and heavy flavours. In section 5 we investigate significant violations of the selection rules, in particular noting that the channels $0^{-+}, 1^{++}, 1^{-+}, 1^{--} \rightarrow \pi + \rho,\ \pi + \gamma,\ KK^*,\ \pi\eta$ may be non-negligible. In the conclusions, section 6, we confront these results with emerging candidates, highlighting the positive and negative features of the VES and BNL $(0, 1, 2)^{-+}$ states, and consider the implications for diffractive photoproduction of vector hybrids and the future extension to hybrid charmonium.

2 General formalism for mesons and their decays

The flux-tube model was motivated by the strong coupling expansion of lattice QCD and to some extent by early descriptions of flux-tubes as cylindrical bags of coloured fields [5, 21, 22]. As application of plaquette operators in lattice QCD extends a line of flux only in the transverse direction, then in the flux-tube model by analogy one allows only locally transverse fluctuations of the tube. There are correspondingly *two* degrees of freedom for excitation of mesons: the relative coordinate $\mathbf{r}_A \equiv \bar{\mathbf{r}} - \mathbf{r}$ of the $\bar{Q} - Q$ and the transverse coordinate of the flux-tube $\mathbf{y}_\perp \equiv -(\mathbf{y} \times \hat{\mathbf{r}}_A) \times \hat{\mathbf{r}}_A$ (see fig. 4a of ref. [15]).

The flux-tube which connects the quark and antiquark may be represented by a system of N beads a distance a apart in modes $\{n_{m+}, n_{m-}\}$ [5] , vibrating w.r.t. the $Q\bar{Q}$- axis $\mathbf{r}_A$ as equilibrium position. The beads are connected to each other and to the quarks at the ends via a non-relativistic string potential with string tension b. The flux-tube has in general Λ units of angular momentum around the $Q\bar{Q}$-axis.

Decay occurs when the flux-tube breaks at any point along its length, producing in the process a $\bar{q}q$ pair in a relative $J^{PC} = 0^{++}$ state. This is similar in spirit to the old Quark Pair Creation or $^3P_0$ model [23] but with an essential difference. In the $^3P_0$ model the $\bar{q}q$ have equal chance of being created at any distance from the initial $\bar{Q}Q$ axis (the "tube" is infinitely thick) whereas in the flux-tube model the distribution of the $^3P_0$ pair *transverse* to the $\bar{Q}Q$ axis is controlled by the transverse ($\mathbf{y}_\perp$) distribution of the flux-tube. For conventional mesons, where the flux-tube is in its ground state, this distribution is parametrised as a Gaussian (eqn. 1 below) such that the tube has a finite width; for hybrid states where the string is excited, the distribution is more structured, in particular containing a node along the initial $\bar{Q}Q$ axis (eqn. 2 below). This gives characteristic constraints on hybrid decay amplitudes, in particular leading to a selection rule against certain hybrid decay modes.

Specifically, the pair creation amplitudes are formulated as follows.

The pair creation position $\mathbf{y}$ [15] is measured relative to the origin (the CM of the initial quarks) and $\mathbf{y}_\perp \equiv -(\mathbf{y} \times \hat{\mathbf{r}}_A) \times \hat{\mathbf{r}}_A$. The $\mathrm{Q\bar{q}}$ - axes of the final states B and C are $\mathbf{r}_B = \mathbf{r}_A/2 + \mathbf{y}$ and $\mathbf{r}_C = \mathbf{r}_A/2 - \mathbf{y}$ respectively. The flux-tube overlap $\gamma(\mathbf{r}_A, \mathbf{y}_\perp)$ (defined below) is assumed to be independent of $\mathbf{y}_{||} \equiv (\mathbf{y} \cdot \hat{\mathbf{r}}_A)\hat{\mathbf{r}}_A$ [15] .

When the flux-tube is in its ground state (*conventional mesons*) the pair creation amplitude is

$$\gamma(\mathbf{r}_A, \mathbf{y}_\perp) = A^0_{00} \sqrt{\frac{fb}{\pi}} \exp(-\frac{fb}{2}\mathbf{y}_\perp^2) \qquad (1)$$

The thickness of the flux tube is related inversely to f (where the infinitely thick flux tube with $f = 0$ corresponds to the $^3P_0$ - model). A detailed discussion of these quantities and the structure of eqn. 1 may be found in ref. [15, eqn. (A21)] and ref. [24].

In this work we consider the decay of the energetically lowest lying *hybrid meson* A with $\{n_{m+}, n_{m-}\} = (n_{1+} = 1\, or\, n_{1-} = 1,\ n_{m\pm} = 0\ \forall m \neq 1)$ and $\Lambda_A = \pm 1$ [5]. The flux-tube overlap [24, 16] prohibits pair creation on the hybrid Q$\bar{\text{Q}}$-axis :

$$\gamma(\mathbf{r}_A, \mathbf{y}_\perp) = \kappa\sqrt{2b}A^0_{00}\sqrt{\frac{fb}{\pi}}\mathbf{e}_\Lambda(\hat{\mathbf{r}}_A)\cdot\mathbf{y}_\perp \exp(-\frac{fb}{2}\mathbf{y}_\perp^2) \tag{2}$$

Here κ is approximately constant [24] and $\mathbf{e}_\Lambda(\hat{\mathbf{r}}_A)$ refers to body-fixed coordinate system.

The entire analysis follows once the full amplitude for the process $A \to BC$ has been formulated. The master equation is eqn. 3 below. Its qualitative structure involves the wavefunctions $\psi(\vec{r})$ for the $\bar{Q}Q$ degrees of freedom of the mesons $\{A; BC\}$, the $\gamma(\mathbf{r}_A, \mathbf{y}_\perp)$ being the flux-tube breaking amplitude and the $\boldsymbol{\sigma}$ and $\boldsymbol{\nabla}$ factors reflecting the $^3P_0$ quantum numbers of the created pair.

To specify the decay amplitude, consider a quark-antiquark bound system A with quark at position $\mathbf{r}$ and antiquark at $\bar{\mathbf{r}}$. The system has a momentum $\mathbf{p}_A$ and wavefunction $\psi_A^{LM_L\Lambda}(\bar{\mathbf{r}} - \mathbf{r})$, with relative coordinate $\mathbf{r}_A \equiv \bar{\mathbf{r}} - \mathbf{r}$, angular momentum L_A and projection M_L^A.

We shall focus on two initial quarks of mass M, with pair creation of quarks of mass m. The decay amplitude $M(A \to BC) \equiv \langle A \mid \hat{\mathcal{C}} \mid BC\rangle$ as obtained from eqns. 35 and 36 (Appendix A) can be shown, in the rest frame of A, to be given by

$$\begin{aligned}
M^{M_L^A M_L^B M_L^C}_{M_S^A M_S^B M_S^C}(A \to BC) &= -\frac{a\tilde{c}}{9\sqrt{3}}(2\pi)^3\delta^3(\mathbf{p}_B + \mathbf{p}_C)\frac{i}{2}Tr(A^T BC)^F Tr(A^T B\boldsymbol{\sigma}^T C)^{SM_S}.\\
&\times \int d^3\mathbf{r}_A\, d^3\mathbf{y}\, \psi_A^{LM_L\Lambda}(\mathbf{r}_A)\exp(i\frac{M}{m+M}\mathbf{p}_B\cdot\mathbf{r}_A)\gamma(\mathbf{r}_A, \mathbf{y}_\perp)\\
&\times (i\boldsymbol{\nabla}_{\mathbf{r}_B} + i\boldsymbol{\nabla}_{\mathbf{r}_C} + \frac{2m}{m+M}\mathbf{p}_B)\psi_B^{LM_L\Lambda *}(\mathbf{r}_B)\psi_C^{LM_L\Lambda *}(\mathbf{r}_C)\ +\ (B \leftrightarrow C)
\end{aligned} \tag{3}$$

where $a\tilde{c}$ is an unknown constant, subsumed into the quantity $\frac{a\tilde{c}}{9\sqrt{3}}\frac{1}{2}A^0_{00}\sqrt{\frac{fb}{\pi}}$ (see Appendix A and also eqn. 16 later) that has been fitted [15] to the known decays of ordinary mesons. Since the lattice spacing a sets the scale in the flux-tube model, it is introduced to obtain the correct dimension for $M(A \to BC)$.

Also $(B \leftrightarrow C)$ indicates a term obtained by interchanging flavour $B^F \leftrightarrow C^F$, spin $B^{SM_S} \leftrightarrow C^{SM_S}$ and momenta $\mathbf{p}_B \leftrightarrow \mathbf{p}_C$ in the first term in eqn. 3. We shall refer to the last two lines in eqn. 3 as the *space part* of $M(A \to BC)$. This is the same for both terms up to an overall sign and so from now on it is sufficient just to consider the exhibited term.

The helicity amplitude can now be constructed from the L-S basis amplitude

$$\begin{aligned}
M_{M_J^A M_J^B M_J^C}(A \to BC) &= \sum_{\{M_L, M_S\}^{A,B,C}} M^{M_L^A M_L^B M_L^C}_{M_S^A M_S^B M_S^C}(A \to BC)\\
&\times \langle L_A M_L^A\, S_A M_S^A \,|\, J_A M_J^A\rangle\, \langle L_B M_L^B\, S_B M_S^B \,|\, J_B M_J^B\rangle\, \langle L_C M_L^C\, S_C M_S^C \,|\, J_C M_J^C\rangle
\end{aligned} \tag{4}$$

To convert to partial wave amplitudes we perform the vector sum of the three total angular momenta $\mathbf{J}_A$, $\mathbf{J}_B$ and $\mathbf{J}_C$ in the order $\mathbf{J} = \mathbf{J}_B + \mathbf{J}_C$ and $\mathbf{J}_A = \mathbf{J} + \mathbf{L}$, and obtain the Jacob-Wick formula [25]

$$M_{LJ}(A \to BC) \;=\; \sqrt{\frac{2L+1}{2J_A+1}} \sum_{\{M_J\}^{A,B,C}} M_{M_J^A M_J^B M_J^C}(A \to BC)$$
$$\times \langle L0\, J M_J^A \mid J_A M_J^A \rangle \langle J_B M_J^B \, J_C M_J^C \mid J M_J^A \rangle \tag{5}$$

The decay rate $\Gamma_{LJ}(A \to BC)$ is calculated according to the prescription of Isgur and Kokoski [15]. Here A, B and C are assumed to be narrow resonances obeying non-relativistic kinematics, while some relativistic effects are taken into account by using masses $\tilde{M}$ as defined in ref. [15] :

$$\Gamma_{LJ}(A \to BC) = \frac{p_B}{(2J_A+1)\pi} \frac{\tilde{M}_B \tilde{M}_C}{\tilde{M}_A} \mid M_{LJ}(A \to BC) \mid^2 \tag{6}$$

Throughout this work, all resonances are assumed to be narrow, and threshold effects are not taken into account.

The decay amplitudes and widths may now be calculated once the wavefunctions $\psi(\vec{r})$ and values of parameters are specified. We shall apply these calculations to a broad class of hybrid decays: the "allowed" couplings to two-body final states consisting of $L=0$ and $L=1$ $Q\bar{q}$ states and the "forbidden" transitions where both of the produced mesons are $L=0$ $Q\bar{q}$-states. In the former class we shall recover the numerical results of Isgur et al. [15] as a particular case. Our analytical results enable extension beyond previous works, in particular to the class of forbidden decays some of which, as we shall see, may be significant and hence offer the prospect of enhanced production.

3 Selection Rules for Hybrid Mesons

The literature contains detailed studies of the decays of *ordinary mesons* both numerically with exact wavefunctions appropriate to a QCD-improved quark model [26] and analytically in an harmonic oscillator approximation [15]. For the case of *hybrid* mesons the rather limited literature reports only numerical results and for a restricted class of "exotic" J^{PC} only [16]. As noted in ref. [15] the analytic forms reveal the relationships that exist among amplitudes as well as establishing the sensitivity of results to parameters. The consistency of the results in the two approaches for the ordinary mesons encourages us to perform an analogous *analytical* calculation of the decay amplitude of an initial energetically lowest lying *hybrid meson* A having $L_A = 1$, with outgoing ordinary mesons B and C ($\Lambda_B = \Lambda_C = 0$) having $L_B = 0$ or 1 and $L_C = 0$. We employ S.H.O. wave functions with inverse radii β_B and β_C for the corresponding states

$$\psi_C^{L=0}(\mathbf{r}) = \mathcal{N}_C \exp(-\beta_C^2 r^2/2) \qquad \psi_B^{L=1}(\mathbf{r}) = \mathcal{N}_B r Y_{1M_L^B}(\hat{\mathbf{r}}) \exp(-\beta_B^2 r^2/2)$$

$$\mathcal{N}_C = \frac{\beta_C^{3/2}}{\pi^{3/4}} \qquad \mathcal{N}_B = 2\sqrt{\frac{2}{3}} \frac{\beta_B^{5/2}}{\pi^{1/4}} \tag{7}$$

For the case of B, C both $L=0$, the $\psi_C(\mathbf{r})$ is to be used with $\beta_C \to \beta_B$ and $\mathcal{N}_C \to \mathcal{N}_B$ as necessary.

For hybrids there is a centrifugal barrier for the $Q\bar{Q}$ pair that arises from the matrix element of $\vec{L}_Q^2$ in the full quark-and-flux-tube angular momentum eigenstate. The angular wavefunction of the combined gluon or flux-tube and quark system was discussed by Horn and Mandula [21] and subsequently by Hasenfratz *et.al.* [22] and by Isgur and Paton [26]. The latter references give essentially the same rigid body angular wavefunction for the full system, which is in the *body-fixed* coordinate system

$$\psi_A^{LM_L^A\Lambda}(\mathbf{r}_A) \sim \mathcal{D}^1_{M_L^A\Lambda}(\phi,\theta,-\phi) \tag{8}$$

This is the amplitude given by Isgur and Paton [5] to find the $Q\bar{Q}$-axis pointing along (θ,ϕ), in a hybrid state with total orbital angular momentum L_A and z-projection M_L^A for a flux-tube with Λ units of angular momentum around the $Q\bar{Q}$-axis. We shall restrict attention to the lowest lying state where $L_A = 1, \Lambda = \pm 1$.

We allow for a general radial dependence of the hybrid wave function parameterized by δ, with $0 < \delta \leq 1$,

$$\psi_A(\mathbf{r}) = \mathcal{N}_A r^\delta \mathcal{D}^1_{M_L^A\Lambda}(\phi,\theta,-\phi)\exp(-\beta_A^2 r^2/2) \qquad \mathcal{N}_A = \sqrt{\frac{3\beta_A^{3+2\delta}}{2\pi\Gamma(3/2+\delta)}} \tag{9}$$

Isgur and Paton [5, eqn. 28] introduced a simple approximation for the matrix element of $\vec{L}_Q^2$ in this state, which neglects a mixing term that raises and lowers Λ. This approximation gives $\vec{L}_Q^2 = L_A(L_A+1) - \Lambda^2$ which transforms the Schrodinger equation into an ordinary differential equation for the (adiabatic) radial wavefunction. Thus, for our case where $L_A = 1, \Lambda = \pm 1$ we can reproduce the small r behaviour of the Schrödinger equation for the hybrid meson by choosing $\delta = 0.62$, satisfying $\delta(\delta+1) = L_A(L_A+1) - \Lambda^2 = 1$. (In practice the values $\delta = 0.62$ or 1 give similar numerical results).

The essential origin of the much advertised selection rules and their violation is driven by the third line of the master equation 3 as we shall now see.

For the case of B and C being $L = 0$ $q\bar{q}$ mesons where both have the wave function $\psi_C(\mathbf{r})$ in eqn. 7, but with $\beta_B \neq \beta_C$ in general, one has

$$\begin{aligned}
&\mathbf{e}_\sigma^* \cdot (i\boldsymbol{\nabla}_{\mathbf{r}_B} + i\boldsymbol{\nabla}_{\mathbf{r}_C} + \frac{2m}{m+M}\mathbf{p}_B)\,\psi_B^*(\mathbf{r}_B)\psi_C^*(\mathbf{r}_C) = \\
&\qquad \mathcal{N}_B\mathcal{N}_C \exp(-\tilde{\beta}^2(\frac{\mathbf{r}_A^2}{4} + \mathbf{y}^2) - \frac{\Delta}{2}\mathbf{r}_A\cdot\mathbf{y})\;\mathbf{e}_\sigma^*\cdot(-i\tilde{\beta}^2\mathbf{r}_A - i\Delta\mathbf{y} + \frac{2m}{m+M}\mathbf{p}_B)
\end{aligned} \tag{10}$$

with the average $\tilde{\beta}^2 \equiv (\beta_B^2 + \beta_C^2)/2$ and difference $\Delta \equiv \beta_B^2 - \beta_C^2$. The nature of the selection rule suppressing the transition to $L = 0$ states arises when we perform the y-integration for which only terms linear in $\mathbf{y}$ contribute (essentially due to the $\mathbf{y}_\perp$ factor in the pair creation amplitude $\gamma(\mathbf{r}_A, \mathbf{y}_\perp)$ eqn. 2), and hence the result is linearly proportional to Δ, the multiplier of of these terms. To the extent that hadrons have the same size, such that $\beta_B = \beta_C$, the integral vanishes and hence the selection rule is immediate (we shall consider the corrections due to $\beta_B \neq \beta_C$ in §5).

By contrast, for the case where the B+C system consists of a $L = 0$ and $L = 1$ $Q\bar{q}$ meson pair the corresponding expression becomes

$$\begin{aligned}
&(i\boldsymbol{\nabla}_{\mathbf{r}_B} + i\boldsymbol{\nabla}_{\mathbf{r}_C} + \frac{2m}{m+M}\mathbf{p}_B)\,\psi_B^*(\mathbf{r}_B)\psi_C^*(\mathbf{r}_C) = \mathcal{N}_C\exp(-\tilde{\beta}^2(\frac{\mathbf{r}_A^2}{4} + \mathbf{y}^2) - \frac{\Delta}{2}\mathbf{r}_A\cdot\mathbf{y}) \\
&\quad \times \mathcal{N}_B[i\sqrt{\frac{3}{4\pi}}\mathbf{e}^*_{M_L^B} + (\frac{r_A}{2}Y^*_{1M_L^B}(\hat{\mathbf{r}}_A) + yY^*_{1M_L^B}(\hat{\mathbf{y}}))\,(-i\tilde{\beta}^2\mathbf{r}_A - i\Delta\mathbf{y} + \frac{2m}{m+M}\mathbf{p}_B)]
\end{aligned} \tag{11}$$

The approximation of equal size gives the leading non-vanishing amplitude in general and allowing $\beta_B \neq \beta_C$ gives corrections. In this first orientation we shall simplify to the approximation $\beta_B = \beta_C$ whence

$$\int d^3\mathbf{y}\gamma(\mathbf{r}_A,\mathbf{y}_\perp) \times (eqn.\,11) = \sqrt{\frac{3}{8\pi}}\mathcal{N}_S\mathcal{N}_B\mathcal{N}_C(-i\tilde{\beta}^2\mathbf{r}_A + \frac{2m}{m+M}\mathbf{p}_B)$$
$$\times \exp(-\frac{\tilde{\beta}^2\mathbf{r}_A^2}{4}) \sum_{M_L'^B} \mathcal{D}^{1\,*}_{M_L^B M_L'^B}(\phi,\theta,-\phi)\bar{\gamma}^1_{M_L'^B\Lambda}$$

$$\bar{\gamma}^1_{M_L'^B\Lambda} \equiv 2\int d^3\mathbf{y}\ \mathbf{e}_\Lambda(\hat{\mathbf{r}}_A)\cdot\mathbf{y}_\perp\ \mathbf{e}^*_{M_L'^B}\cdot\mathbf{y}\ \exp(-\tilde{\beta}^2\mathbf{y}^2 - \frac{fb}{2}\mathbf{y}_\perp^2) \tag{12}$$

where we used $yY^*_{1M_L'^B}(\hat{\mathbf{y}}) = \sqrt{\frac{3}{4\pi}}\mathbf{e}^*_{M_L'^B}\cdot\mathbf{y}$, and defined $\mathcal{N}_S \equiv \kappa\sqrt{b}A^0_{00}\sqrt{\frac{fb}{\pi}}$.

The angular momentum projection M_L^B is defined relative to the *space-fixed axes* (with $\mathbf{p}_B$ defining the $\hat{\mathbf{z}}$-axis), as usual. The y-integration is done in the system of *body-fixed axes* (with the q$\bar{\text{q}}$-axis defining the $\hat{\mathbf{z}}$-axis) and so we must convert to angular momentum projection $M_L'^B$ relative to the body-fixed system. The body is moving with its $\hat{\mathbf{z}}$-axis rotated by rotation matrix $\mathcal{R}$ relative to the space-fixed coordinate system, i.e. $|\,\psi(\hat{\mathbf{y}}_\mathbf{r})\rangle = \mathcal{R}\,|\,\psi(\hat{\mathbf{y}}_{r\hat{\mathbf{z}}})\rangle$. The spherical harmonics transform as

$$Y_{LM}(\hat{\mathbf{y}}_\mathbf{r}) = \langle\psi(\hat{\mathbf{y}}_\mathbf{r})\,|\,\psi(rLM)\rangle = \sum_{M'}\langle\psi(\hat{\mathbf{y}}_{r\hat{\mathbf{z}}})\,|\,\psi(rLM')\rangle\langle\psi(rLM')\,|\,\mathcal{R}^+\,|\,\psi(rLM)\rangle$$
$$= \sum_{M'} Y_{LM'}(\hat{\mathbf{y}}_{r\hat{\mathbf{z}}})\mathcal{D}^L_{MM'}(\phi,\theta,-\phi) \tag{13}$$

where we used $\mathcal{D}^L_{MM'}(\phi,\theta,-\phi) \equiv \langle\psi(rLM')\,|\,\mathcal{R}^+\,|\,\psi(rLM)\rangle$ [5, Appendix A]. Performing the $\mathbf{y}$ integration in eqn. 12

$$\bar{\gamma}^1_{M_L'^B\Lambda} = \frac{\pi^{3/2}}{\tilde{\beta}(\tilde{\beta}^2+fb/2)^2}\delta_{M_L'^B\Lambda} \equiv \bar{\gamma}^1\,\delta_{M_L'^B\Lambda} \tag{14}$$

and hence there is an important selection rule operating in the moving frame of the initial $Q\bar{Q}$-pair : The one unit of angular momentum of the incoming hybrid around its $Q\bar{Q}$-axis is exactly absorbed by the component of the angular momentum of the outgoing meson B along this axis. This helps to generate relationships among amplitudes for decays into $L=0$ and $L=1$ $Q\bar{q}$ states.

4 Hybrid meson decay into $L=1$ and pseudoscalar mesons

The overall strengths follow once integration over $\mathbf{r}_A$ is performed. In the harmonic oscillator basis the amplitudes can be calculated analytically or at least reduced to tractable forms that expose their detailed structure and parametric dependences.

Expanding $\exp(i\frac{M}{m+M}\mathbf{p}_B\cdot\mathbf{r}_A)$ in spherical Bessel functions and Legendre polynomials we obtain for the space part of the decay amplitude in eqn. 3 (using eqns. 12 and 14)

$$M(A \to BC) = \sqrt{\frac{3}{8\pi}} \mathcal{N}_A \mathcal{N}_B \mathcal{N}_C \mathcal{N}_S \bar{\gamma}^1 \sum_{n=0}^{\infty} (2n+1) i^n$$
$$\times \int_0^\infty dr_A r_A^{2+\delta} j_n(\frac{M}{m+M} p_B r_A) \exp(-(2\beta_A^2 + \tilde{\beta}^2)\frac{r_A^2}{4})$$
$$\times \int d\Omega_{\hat{\mathbf{r}}_A} (-i\tilde{\beta}^2 \mathbf{r}_A + \frac{2m}{m+M} \mathbf{p}_B) \mathcal{D}^1_{M_L^A \Lambda}(\phi,\theta,-\phi) \mathcal{D}^{1\,*}_{M_L^B \Lambda}(\phi,\theta,-\phi) P_n(cos\theta) \tag{15}$$

The partial wave amplitudes $M_L(A \to BC)$ (with $J = J_B$) of eqn. 5 can now be evaluated. We obtain (modulo the factor $(2\pi)^3 \delta^3(\mathbf{p}_B + \mathbf{p}_C)$)

$$M_L(A \to BC) = (\frac{a\tilde{c}}{9\sqrt{3}} \frac{1}{2} A^0_{00} \sqrt{\frac{fb}{\pi}}) \frac{\kappa\sqrt{b}}{(1 + fb/(2\tilde{\beta}^2))^2} \sqrt{\frac{2\pi}{3\Gamma(3/2+\delta)}} \frac{\beta_A^{3/2+\delta}}{\tilde{\beta}} \tilde{M}_L(A \to BC) \tag{16}$$

In Table 1 we display the reduced partial wave amplitudes $\tilde{M}_L(A \to BC)$ in a compact form by defining

$$\begin{array}{lll} S = -(3\tilde{h}_0 - \tilde{g}_1 + 4\tilde{h}_2) & P_1 = -i(2\tilde{g}_0 + 3\tilde{h}_1 - \tilde{g}_2) & P_2 = -i(\tilde{g}_0 + \tilde{g}_2) \\ P_3 = -i(5\tilde{g}_0 + 3\tilde{h}_1 + 2\tilde{g}_2) & P_4 = -i(10\tilde{g}_0 + 9\tilde{h}_1 + \tilde{g}_2) & P_5 = -i(5\tilde{g}_0 + 6\tilde{h}_1 - \tilde{g}_2) \\ D = (\tilde{g}_1 + 5\tilde{h}_2) & F = -3i(\tilde{g}_2 + \tilde{h}_3) & G = 0 \end{array} \tag{17}$$

$$\begin{pmatrix} \tilde{g}_n \\ \tilde{h}_n \end{pmatrix} = \begin{pmatrix} \frac{2m}{m+M}\mathbf{p}_B \\ \tilde{\beta}^2 \end{pmatrix} \int_0^\infty dr \begin{pmatrix} 1 \\ r \end{pmatrix} r^{2+\delta} j_n(\frac{M}{m+M} p_B r) \exp(-(2\beta_A^2 + \tilde{\beta}^2)\frac{r^2}{4}) \tag{18}$$

For $\delta = 1$ explicit evaluation of $\tilde{g}_1$, $\tilde{h}_0$ and $\tilde{h}_2$ can be made using

$$\int_0^\infty dr \begin{pmatrix} j_1(ur) \\ rj_0(ur) \\ rj_2(ur) \end{pmatrix} r^3 \exp(-wr^2) = \frac{\sqrt{\pi}}{16 w^{7/2}} \exp(-\frac{u^2}{4w}) \begin{pmatrix} 2uw \\ 6w - u^2 \\ u^2 \end{pmatrix} \tag{19}$$

The only free parameter in the model is the overall normalization of decays subsumed in κ and the the combination $\frac{a\tilde{c}}{9\sqrt{3}}\frac{1}{2}A^0_{00}\sqrt{\frac{fb}{\pi}}$ in eqn. 16. However, if one repeats the analysis of §3 but with the hybrid decay amplitude (eqn. 2) replaced by that appropriate to ordinary mesons (eqn. 1), one finds that the same dimensionless combination controls the (known) decays of the conventional mesons. A best fit gave a value of 0.64 [15] and we adopt this accordingly. The scale of hybrid decays relative to ordinary meson decays are then determined by κ: however, in the simplified framework of ref. [24] the estimated values for $N = 3 - 5$ beads are $f = 1.1$, $\kappa = 0.9$ and $A^0_{00} = 1.0$.

Our analytical calculation (with simplified wave functions) reproduces an earlier numerical computation for light hybrids with exotic J^{PC} [16] to within 15% on average. If we use the same hadron masses as ref. [16], follow their prescription (as outlined in ref. [15]) of ignoring all quark flavour symmetry breaking and normalizing the decays as above, we find that the optimal comparison with ref. [16] follows with $\beta_A = 0.27$ GeV and $\tilde{\beta} = 0.28$ GeV throughout: this gives the widths in Table 2. We confirm their result that the decays indicated are dominant, except for the case $J^{PC} = 0^{+-}$ where we find also prominent decays $(\pi,\omega)0^{+-} \to K_1(1270)K$ (with width 400 MeV) and $\pi 1^{-+} \to K_1(1400)K$ (with width 100 MeV) which were not listed in ref. [16].

Our analysis provides an independent check on the results of ref. [16] and enables us to examine their sensitivity to the parameters. This merits attention since the best fit to the widths of conventional mesons by [15] used a rather different value for β, namely $\beta_A = \tilde{\beta} = 0.4$. Indeed, this is in line with the modern preferred values from harmonic oscillator basis approximations to meson spectroscopy e.g. in the ISGW work [18]. Our preferred choice today is to adopt the harmonic oscillator basis fit to spin-averaged meson spectroscopy of ref. [18]. Wherever values for β are not available, we abstract them from the mean meson radii of Merlin [27]. We take the string tension $b = 0.18\, GeV^2$, and the constituent-quark masses $m_u = m_d = 0.33$ GeV, $m_s = 0.55$ GeV and $m_c = 1.82$ GeV. Meson masses are taken from ref. [28], and where not available (as in the case of ${}^3P_1/{}^1P_1$ mixing angles) we abstract them from spectroscopy predictions [26] suitably adjusted relative to known masses. Hybrid β's, masses before spin splitting and hyperfine splittings derive from Merlin [27, 29]. Our quoted widths are computed for $\delta = 0.62$ (though as mentioned earlier, the results with $\delta = 1$ are essentially similar to these).

We are able here, for the first time, to present also the most prominant predicted widths for both exotic and non-exotic J^{PC} combinations. These are displayed for u, d, s flavours in tables 3 - 5 together with the values assumed for parameters. One can choose alternative values for these parameters and modify the widths accordingly by use of table 1 and eqns. 16 - 19.

Application to hybrid charmonium decays $c\bar{c}\; hybrid \to D^{**}D$ follows rather directly. Their masses are predicted in the flux-tube model to be ≈ 4.3 GeV [6] which is in the vicinity of the $D^{**}D$ threshold. It is possible therefore that hybrid charmonium will be kinematically forbidden from decaying into the preferred $(L = 0)$ + $(L = 1)$ ($\{D$ or $D^*\}$ + D^{**}) states, in which case their widths may be narrow and their signals enhanced through decays into $\psi, \psi' \cdots$ [14]. Studies of hybrid charmonia will be reported elsewhere [19].

5 Hybrid meson decay into two $L = 0$ mesons

For decays of hybrid mesons into two $L = 0$ $q\bar{q}$ mesons, the flux-tube model predictions are very distinctive. When $\beta_B = \beta_C$ the hybrid decay width is zero because the one unit of angular momentum of the incoming hybrid around the $Q\bar{Q}$-axis cannot be absorbed by the angular momenta of the outgoing mesons. Non-zero widths arise if the S-wave hadrons have different size (a result originally noted in the ${}^3P_0$ limit in ref. [23]).

Inserting eqn. 2 into the master equation 3 and performing the y-integration, only terms linear in $\mathbf{y}$ in eqn. 10 contribute

$$\int d^3\mathbf{y}\gamma(\mathbf{r}_A, \mathbf{y}_\perp) \times (eqn.\, 10) = -\frac{i\Delta}{\sqrt{2}} \mathcal{N}_S \mathcal{N}_B \mathcal{N}_C \bar{\gamma}^1 \exp(-\frac{\tilde{\beta}^2 \mathbf{r}_A^2}{4} + (\frac{\mathbf{r}\Delta}{4\tilde{\beta}})^2\,)\, \mathcal{D}^{1\,*}_{\sigma\Lambda}(\phi, \theta, -\phi) \quad (20)$$

where $\bar{\gamma}^1$ is defined in eqn. 14, and the y-integration is done in the body-fixed system, introducing an extra $\mathcal{D}$-function as in section 3. Clearly the decay amplitude is proportional to $\Delta \equiv \beta_B^2 - \beta_C^2$: when $\beta_B = \beta_C$, decay is prohibited. Nonetheless, it is instructive to present the results scaled by the factor Δ. As we shall see, some of the widths would be substantial were it not for this factor and hence it will be important to consider the implications of a small, non-zero, value for Δ in hybrid meson phenomenology.

We perform the r-integration in eqn. 3 as in section 4. The partial wave amplitudes $M_L(A \to BC)$ are (modulo the factor $(2\pi)^3 \delta^3(\mathbf{p}_B + \mathbf{p}_C)$)

$$M_L(A \to BC) = -(\frac{a\check{c}}{9\sqrt{3}}\frac{1}{2}A^0_{00}\sqrt{\frac{fb}{\pi}})\frac{\kappa\sqrt{b}}{(1+fb/(2\tilde{\beta}^2))^2} \tag{21}$$
$$\times\,\Delta\sqrt{\frac{\pi}{3\Gamma(3/2+\delta)}}\frac{\beta_A^{3/2+\delta}(\beta_B\beta_C)^{3/2}}{\tilde{\beta}^5}\check{M}_L(A \to BC)$$

In Table 6 we display the reduced partial wave amplitudes in a compact form by defining

$$S = \check{g}_0 \quad P = -i\check{g}_1 \quad D = \check{g}_2 \quad F = 0 \quad G = 0 \tag{22}$$

$$\check{g}_n = \int_0^\infty dr r^{2+\delta} j_n(\frac{M}{m+M}p_B r)\exp(-(2\beta_A^2 + \tilde{\beta}^2 - (\frac{\Delta}{2\tilde{\beta}})^2)\frac{r^2}{4}) \tag{23}$$

In Tables 7-9 we display a selection of the most prominent widths calculated from $M_L(A \to BC)$ in eqn. 21, and scaled by the dimensionless ratio

$$(\frac{\Delta}{2\tilde{\beta}^2})^2 = (\frac{\beta_B^2 - \beta_C^2}{\beta_B^2 + \beta_C^2})^2 \tag{24}$$

We define the *intrinsic* width Γ_R by $\Gamma_R(A \to BC) \times (eqn.\,24) = \Gamma(A \to BC)$.

In all cases the same parameter values as in the corresponding flavour modes in §4 and tables 3 - 5 are used. The β's of §4 are the same within the same hyperfine multiplet and so would cause all widths in this section to be zero. However, estimates of β's differing in the same hyperfine multiplet can be found in the literature and these will lead to a non-zero value for Δ and hence non-zero widths.

It is clear from tables 7 - 9 that some of the widths would be substantial were Δ non-zero. In some potential decay channels we would expect $\Delta \neq 0$, for example, the π is anomalously light and may be expected to have an effective β that differs significantly from that of the ρ. Indeed, the "effective" β in ref. [15, Table I] are used in tables 7 - 9 to determine the widths Γ, and give

$$(\frac{\Delta}{2\tilde{\beta}^2})^2 = 0.2\,(\pi\rho);\ 0.14\,(KK^*);\ 0.04\,(DD^*) \tag{25}$$

Similar results obtain in the MIT Bag model [8, fit II] by assuming $1/\beta \propto$ *the bag radius*. In this context, note that the *intrinsic* widths $\Gamma_R(A \to BC)$ are often predicted to be substantial, e.g. for decays into $\pi + \rho$; $\pi + \omega$ and KK^*. Indeed, values of $\sim 30\%$ larger arise for $\rho\pi$ and $\omega\pi$ if one takes an alternative assumption within the MIT Bag dynamics where $\beta \propto m^{-1/3}$ (for massless quarks), but even with the more conservative assumptions of eqn. 25 we see that we anticipate significant couplings of hybrids in several of the "forbidden" channels.

It is possible therefore that hybrids could give rather distinctive signatures in diffractive photoproduction or e^+e^- annihilation, namely the production of vector mesons in $\pi\rho$, $\pi\omega$, KK^* or even DD^* channels, but absent (apart from mixing with conventional quarkonia) in the corresponding $\pi\pi$, $\rho\rho$,$K\bar{K}$, $D\bar{D}$ etc. final states.

6 Phenomenology and Conclusions

1^{-+}

The most obvious signature for a hybrid meson is the appearance of a flavoured state with an exotic combination for J^{PC}. Ref. [9] may have indications for such a state with $J^{PC} = 1^{-+}$ whose mass and decay characteristics are in line with historical expectations. The search was motivated by the selection rule (section 3) and concentrated on the classic decay channel for $S + P$, namely $\pi + f_1$, which is where the candidate has been sighted. The experiment sees a broad structure in the mass region $1.6 - 2.2$ GeV which is suggestive of being a composite of two objects at 1.7 and 2.0 GeV. It is the latter that appears to have a resonant phase though they admit that more data is required for a firm conclusion.

Our expectations for widths from tables 3 and 7 for $J^{PC} = 1^{-+}$ at a mass of 2.0 GeV (which is essentially as originally predicted) are (in MeV)

$$\pi f_1 : \pi b_1 : \pi\rho : \eta\pi : \eta'\pi \;=\; 60 : 170 : 5-20 : 0-10 : 0-10 \tag{26}$$

The former pair are similar to those in ref. [16] but we note also the possible presence of $\pi\rho$ or even $\pi\eta$ decays that are not negligible relative to the signal channel πf_1. This may be important in view of a puzzle, commented upon in ref. [9], that the production mechanism appeared not to be as expected given the anticipated hybrid dynamics. Instead of b_1-exchange, leading to the classic $S+P \quad \pi + b_1(1235)$ coupling, significant $\pi + \rho$ coupling may be responsible. In view of our analysis in §5, and eqn. 26 above, it is clear that the latter coupling may be significant on the scale of the πf_1 signal; the final state decays into $\pi + \rho$ should therefore also be investigated experimentally.

0^{-+}

If this *prima facie* signal is indeed a resonant 1^{-+} hybrid excitation then one expects partners, in particular 0^{-+}, to be in this mass region. The VES Collaboration sees an enigmatic and clear 0^{-+} signal in diffractive production with 37 GeV incident pions on beryllium [10]. They study the channels $\pi^- N \to \pi^-\pi^+\pi^- N$; $\pi^- K^+ K^- N$ and see a resonant signal $M \approx 1790$ MeV, $\Gamma \approx 200$ MeV in the classic $(L = 0) + (L = 1)$ $\bar{Q}q$ channels $\pi^- + f_0$; $K^- + K_0^*$, $K(K\pi)_S$ with no corresponding strong signal in the allowed $L = 0$ two body channels $\pi + \rho$; $K + K^*$. The width and large couplings to kaons are both surprising if this were the second radial excitation of the pion (the first radial excitation is seen as a broad enhancement in accord with expectations). Furthermore, the apparent preference for decay into $(L = 0) + (L = 1)$ mesons at the expense of $L = 0$ pairs is qualitatively in accord with expectations for hybrids.

Our quantitative estimates on the relative importance of available channels further support this identification. For a 0^{-+} hybrid at 1.8 GeV we find widths $\pi f_0(1300) \sim 170$ MeV; $\pi f_2 = 5 - 10$ MeV. The KK_0^* channel which is predicted to dominate for a 2.0 GeV initial state (table 3) is kinematically suppressed though probably non-zero due to the $\sim$ 300 MeV width of the $K_0^*(1430)$. The decay to $L = 0$ pairs, which is naively expected to be suppressed, turns out to be potentially significant, $\pi\rho \sim 30$ MeV for a 1.8 GeV 0^{-+} hybrid. This is compatible with the experimental limit

$$\frac{0^{-+} \to \pi^-\rho^0}{0^{-+} \to \pi^- f_0(1300)} \;<\; 0.3 \; (95\% \; C.L.) \tag{27}$$

The $KK^*(890)$ channel, by contrast, is expected to be a mere ~ 5 MeV, which is consistent with the observed order of magnitude suppression observed in ref. [10]

$$\frac{0^{-+} \to K^- K^*}{0^{-+} \to (K^- K^+ \pi)_S} \ < \ 0.1 \ (95\% \ C.L.) \tag{28}$$

The $\Gamma_{total} \sim 200 - 350$ MeV is also consistent with the observed 200 ± 50 MeV. However, this may be fortuitous. First, the overall scale of widths for hybrids, controlled by the breaking of the excited flux-tube, may differ from that of the ground state conventional decays such that all hybrid predictions will need to be rescaled by an overall constant. Furthermore our calculations are all in the narrow width approximation whereas the $f_0(1300)$ at least is a broad ill defined structure. The precise role of the enigmatic $f_0(980)$, to which the resonance also appears to couple experimentally, also perturbs a detailed analysis at this stage. The data here are

$$\frac{0^{-+} \to \pi^- f_0(980)}{0^{-+} \to \pi^- f_0(1300)} \ = \ 0.9 \pm 0.1 \tag{29}$$

As noted in ref. [10] this is an unexpectedly high value since the $f_0(980)$ has a small width and strong coupling to strangeness while the $f_0(1300)$ is a broad object coupled mainly to non-strange quarks. However, this may be natural for a hybrid at this mass for the following reason. The strongest predicted decay path (see table 3) would be $0^{-+} \to KK_0^*$ but for the fact that this is below threshold for the 1.8 GeV initial state, thus the $(KK\pi)_S$ is expected to be significant (as observed [10]) and, at some level, may feed the channel $\pi f_0(980)$ through the strong affinity of $K\bar{K} \to f_0(980)$. Thus the overall expectations are in line with the data. Important tests are now that there should be a measureable coupling to the $\pi\rho$ channel with only a small πf_2 or KK^* contribution.

$\underline{2^{-+}}$

This suppression of πf_2 for the 0^{-+} is quite opposite to the prediction for the 2^{-+} partner for which this channel should dominate significantly over the πf_0 partner (not least because of the interchanged role of S and D waves). This is a problem if one wishes to identify the 2^{-+} seen at ~ 2.2 GeV at VES as the hybrid partner of the 0^{-+}. The putative signal is claimed in $\pi f_0(1300)$ whereas no πf_2 nor $\pi f_0(980)$ are reported. The properties and existence of this state are less clearcut experimentally and on mass alone it could qualify either as a radial excitation or tantalisingly in accord with the emergence of a family of hybrids. However, as alluded to above, its decay channels do not appear superficially to be in line with those expected for hybrids. The πf_0 is predicted to be small while that to πf_2, πb_1 together with KK^* or πa_2 provide the anticipated signals. From the regularities in table 1 we see immediately the source of the pattern

$$\pi f_0 : \pi f_2 \ = \ 1 : 7 \tag{30}$$

for the D-waves, let alone the S-wave contribution for πf_2. If $\pi f_0(1300) \geq \pi f_2(1270)$ is sustained for this state, it is either not a hybrid or there is some new dynamics connecting it to the broad $f_0(1300)$ state.

Historically the ACCMOR Collaboration [31] has argued for a 2^{-+} state around 2.1 GeV, or possibly 1.8 GeV, coupled to πf_2, which was used to set the mass scale in a Bag Model simulation of hybrids in ref. [8]. The lower mass is tantalisingly similar to sightings of a possible 2^{-+} in photoproduction via π exchange [32] and coupled to $\pi\rho$ and πf_2. These suggest that there may be interesting activity in the 2^{-+} wave which may herald new degrees of freedom; if hybrid components are present in this (these?) state, we urge a search for the πb_1 decay channel which, at the lower mass, could have a branching ratio of up to 50%.

$\underline{1^{--}}$

If these are indeed signalling the emergence of the lowest lying families of hybrids, then there must be a nonet of 1^{--} partners. As the 0^{-+} appeared in diffractive π production, so we anticipate the appearance of the 1^{--} in diffractive photoproduction or e^+e^- annihilation. We advocate searching for the lightest vector hybrids in

$$\gamma(p) \to \pi a_1(p) \to 4\pi(p) \tag{31}$$

where within our harmonic oscillator approximation we predict for an isovector (in MeV)

$$\pi a_1 : \pi a_2 : \pi h_1 : \rho\rho : \pi\omega : \pi\pi \ = \ 170 : 50 : 0 : 0 : 10-20 : 0 \tag{32}$$

Alternatively, for a vector hybrid at a mass of ~ 1.5 GeV (see below) these become

$$\pi a_1 : \pi a_2 : \pi h_1 : \rho\rho : \pi\omega : \pi\pi \ = \ 140 : \sim 0 : 0 : 0 : 5-10 : 0 \tag{33}$$

These are very different from the predictions of radial or $^3D_1$ decays of quarkonia [15, 26, 33]. In particular the suppression of πh_1 relative to πa_1 is, within the flux-tube model, a crucial test of the hybrid initial state in contrast to the case of a $^3D_1$ or radially excited 1^{--} for which the πh_1 would be expected to dominate over πa_1 [15, 33]. The reason is that in the hybrid 1^{--} the $Q\bar{Q}$ have $S=0$, whereas for the "conventional quarkonium" 1^{--} the $Q\bar{Q}$ have $S=1$; the $^3P_0$ decay is forbidden by spin orthogonality in the former example for final states where the mesons' $Q\bar{Q}$ have $S=0$, as in the πh_1 example. It is therefore interesting that the detailed analyses of refs. [12, 30] comment on the apparently anomalous decays that they find for the 1^{--} state "ρ_1"(1450), in particular the suppression of πh_1 relative to πa_1 and the dominance of the latter over the $\pi\omega$:

$$\pi a_1 : \pi h_1 + \rho\rho : \pi\omega : \pi\pi \ = \ 190 : 0-39 : 50-80 : 17-25 \tag{34}$$

It is noticeable that the $\pi\pi$ decay also is strongly suppressed though non-zero; if this is substantiated it could indicate either a deviation from the harmonic wave function approximation or in addition some mixing between hybrid and radial vector mesons in this region. The latter could also rather naturally explain the enhancement of the $\pi\omega$ channel as well as the repulsion of the eigenstate to low mass. This is beyond the present work but merits further attention in view of the fact that the decay channels of the ρ_1, in particular the large πa_1 component and suppressed $\pi\pi$, require that "mixing with non $Q\bar{Q}$ states must occur" [12][1]. We suggest that a detailed comparison of e^+e^- with diffractive photoproduction may help to isolate the hybrid contributions more clearly as the relative abundance of hybrid excitation and quarkonium production is in general expected to differ in the two cases: as diffractive photoproduction involves the transition $\gamma \to Q\bar{Q}$ in the probable presence of a gluonic Pomeron, there is the possibility of "plucking the string".

[1]It is interesting that there appear to be possible solutions to the data with $\pi\pi$ even more suppressed and the πa_1 increased in compensation (A. Donnachie, private communication)

Ref. [12] also finds evidence for $\omega(1440)$ with no visible decays into πb_1 which is in significant contrast to the expectations for conventional $q\bar{q}(^3S_1)$ or $^3D_1$ initial states. In the hybrid interpretation this suppression is natural and is the isoscalar analogue of the πh_1 selection rule alluded to above. It is also interesting to note that for a hybrid $\omega(1440)$, the "wannabee" $(L = 0) + (L = 1)$ decay paths are kinematically suppressed leaving the $\pi\rho$ and possibly $\eta\omega$ as dominant decays.

Insofar as $L = 0$ pairs are predicted to be suppressed but not totally absent in the decay products, searches for $\pi\rho$; KK^*; $\pi\omega\,(\pi\eta)$ should be made. Confirmation of signals in such channels together with them being dominated by $(L = 0) + (L = 1)$ states would add considerable weight to the hybrid hypothesis. We need more detailed study of decays of radial excitations in the quark model to see if they imitate the hybrid preferences for $S + P$ modes: as noted above for the 1^{--} channel, the relative branching ratios to these can be distinctive as in the case of $\pi h_1 : \pi a_1$ which differ appreciably for ρ_{hybrid} and $\rho_{conventional}$. If these are hybrid states then necessarily there will be partners whose production and decay channels become rather tightly constrained.

To the extent that signals are appearing in the expected mass region for light flavours, together with hints of a rich 0^{++} spectroscopy in the mass region anticipated for gluonic excitations in the pure gauge sector, we have increasing confidence in predictions for the gluonic excitations in more generality, in particular for hybrids containing heavy flavours e.g. $c\bar{c}$ and $b\bar{b}$. These are predicted to occur in the vicinity of charm threshold [6, 13] and so we advocate intensive study of this region, in particular with e^+e^-. Rather clear signals and the clean environment may distinguish radial from hybrid here. The $S + S$ suppression is more dramatic than for light flavours and so there is the exciting possibility that hybrid charmonium will be narrow ($\sim 1 - 10$ MeV). Appearance of states above charm threshold decaying into DD^* but strongly suppressed or even absent in $D\bar{D}$, $D^*\bar{D}^*$ would be rather striking.

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8 Appendix A : General decay formalism

Consider a quark-antiquark bound system A with quark at position $\mathbf{r}$ and antiquark at $\bar{\mathbf{r}}$ with masses m and $\bar{m}$ respectively. The system has a momentum $\mathbf{p}_A$ and wavefunction $\psi_A^{LM_L\Lambda}(\bar{\mathbf{r}} - \mathbf{r})$, with relative coordinate $\mathbf{r}_A \equiv \bar{\mathbf{r}} - \mathbf{r}$, angular momentum L_A and projection M_L^A. The flux-tube has Λ units of angular momentum around the q$\bar{\text{q}}$-axis. Introduce a second-quantized formalism in which the normalized wavefunction is written in the L-S basis as

$$|\,A\rangle \equiv |\,A^{CF\,SM_S\,LM_L\Lambda}_{\{n_{m+},n_{m-}\}}(\mathbf{p}_A)\rangle = \sum_{f\bar{f}s\bar{s}c\bar{c}} \int d^3\mathbf{r}\, d^3\bar{\mathbf{r}}\, A^C_{c\bar{c}} A^F_{f\bar{f}} A^{SM_S}_{s\bar{s}} \psi_A^{LM_L\Lambda}(\mathbf{r}_A)$$

$$\times \exp(i\mathbf{p}_A \cdot \frac{m\mathbf{r} + \bar{m}\bar{\mathbf{r}}}{m + \bar{m}})\; q^+_{cfs}(\mathbf{r})\; \bar{q}^+_{\bar{c}\bar{f}\bar{s}}(\bar{\mathbf{r}})\; |0\rangle \otimes\; |\mathbf{r}_A\, \{n_{m+}, n_{m-}\}\rangle \tag{35}$$

with A^C, A^F and A^{SM_S} referring to the colour, flavour and spin matrices respectively. Here $q^+_{cfs}(\mathbf{r})$ and $\bar{q}^+_{cfs}(\mathbf{r})$ are the non-relativistic position space quark and antiquark creation operators respectively, obeying anticommutation relations of the type $\{q^+_{cfs}(\mathbf{r}), q^+_{\bar{c}\bar{f}\bar{s}}(\bar{\mathbf{r}})\} = \delta^3(\mathbf{r}-\bar{\mathbf{r}})\,\delta_{c\bar{c}}\delta_{f\bar{f}}\delta_{s\bar{s}}$. The state $|\,\mathbf{r}_A\,\{n_{m+},n_{m-}\}\rangle$ represents the system of N beads a distance a apart in modes $\{n_{m+},n_{m-}\}$ [5] , vibrating w.r.t. the q$\bar{\text{q}}$ - axis $\mathbf{r}_A$ as equilibrium position. The beads are connected to each other and the quarks at the ends via a non-relativistic string potential with string tension b.

The $^3P_0$ quark-antiquark creation operator $\hat{\mathcal{C}}$ motivated from the strong coupling expansion of Hamiltonian lattice gauge theory [15] is

$$\begin{aligned}\hat{\mathcal{C}} &= \frac{a\tilde{c}}{9}\sum_{cfs\bar{s}}\int d^3\mathbf{x}\; b(\mathbf{x})\psi^+_{cfs}(\mathbf{x})\boldsymbol{\alpha}_{s\bar{s}}\cdot\boldsymbol{\nabla}\psi_{cf\bar{s}}(\mathbf{x}) \\ &= \frac{a\tilde{c}}{9}\sum_{cfs\bar{s}}\int d^3\mathbf{x}\; b(\mathbf{x})q^+_{cfs}(\mathbf{x})\boldsymbol{\sigma}_{s\bar{s}}\cdot\boldsymbol{\nabla}\bar{q}^+_{cf\bar{s}}(\mathbf{x}) \qquad (36)\end{aligned}$$

where we restrict $\hat{\mathcal{C}}$ to q$\bar{\text{q}}$-creation in the last line. Here $\psi_{cfs}(\mathbf{x})$ is the usual relativistic Dirac fermion operator with $\boldsymbol{\alpha}$ the Dirac matrices defined as usual in terms of the Pauli matrices $\boldsymbol{\sigma}$. A bead is annihilated by $b(\mathbf{x})$ at the pair creation position. The identity can be established by defining $q^+_{cfs}(\mathbf{x})$ i.t.o. the quark creation operators of the momentum space expansion of $\psi_{cfs}(\mathbf{x})$. The factor of 1/9 arises by requiring the annihilated flux to couple to a singlet and be unoriented [15]. We introduce an unknown constant $\tilde{c}$ and lattice spacing a, so that $\dim(\hat{\mathcal{C}}) = 1$, as required.

We can now rigorously define the flux-tube overlap $\gamma(\mathbf{r}_A,\mathbf{y}_\perp) \equiv \langle \mathbf{r}_A\,\{n_{m+},n_{m-}\}\,|\,b(\mathbf{y})\,|\,\mathbf{r}_B\mathbf{r}_C\,\{n_{m+},n_{m-}\}\rangle$ introduced in §2.

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Table 1: Partial wave amplitudes $\tilde{M}_L(A \to BC)$ written in terms of the functions defined in eqn. 17 and named in accordance with partial wave S, P, D, F or G. We display various J^{PC} of the initial hybrid A decaying into a L=1 meson B and pseudoscalar meson C. Starred amplitudes vanish even with non-S.H.O. radial wave functions.

A	B	$\tilde{M}_L$	A	B	$\tilde{M}_L$	A	B	$\tilde{M}_L$
2^{-+}	2^{++}	$-\sqrt{5}S/\sqrt{18}$	1^{-+}	2^{++}	$0 \times D$	1^{+-}	2^{++}	$-P_3/\sqrt{15}$
		$-\sqrt{7}D/3$		1^{++}	$S/\sqrt{6}$			$-F/\sqrt{10}$
		G			$-D/\sqrt{3}$		1^{++}	P_2
	1^{++}	$0 \times D$		1^{+-}	$S/\sqrt{3}$		0^{++}	$P_1/\sqrt{3}$
	0^{++}	$D/3$			$D/\sqrt{6}$		1^{+-}	$-P_1/\sqrt{2}$
	1^{+-}	$-D/\sqrt{2}$	0^{-+}	2^{++}	$D/3$	0^{+-}	1^{++}	$-P_1/\sqrt{3}$
2^{+-}	2^{++}	$P_5/\sqrt{5}$		0^{++}	$\sqrt{2}S/3$		1^{+-}	$\sqrt{2}P_2/\sqrt{3}$
		$-F/\sqrt{5}$	1^{--}	2^{++}	$D/\sqrt{2}$	1^{++}	2^{++}	$-P_4/\sqrt{30}$
	1^{++}	$P_3/\sqrt{15}$		1^{++}	$S/\sqrt{3}$			$F/\sqrt{5}$
		$F/\sqrt{10}$			$D/\sqrt{6}$		1^{++}	$-P_1/\sqrt{2}$
	1^{+-}	$P_4/\sqrt{30}$		1^{+-}	$0 \times S^*$		0^{++}	$-\sqrt{2}P_2/\sqrt{3}$
		$-F/\sqrt{5}$			$0 \times D^*$		1^{+-}	$0 \times P^*$

Table 2: Widths in MeV for *hybrid* $A \to BC$ for exotic hybrid J^{PC} in partial wave L. Here π, ω and ϕ indicate flavour states $\sqrt{\frac{1}{2}}(u\bar{u} - d\bar{d})$, $\sqrt{\frac{1}{2}}(u\bar{u} + d\bar{d})$ and $s\bar{s}$ respectively. We adopted hybrid masses of 1.9 GeV (π, ω) and 2.1 GeV (ϕ); a ${}^3P_1/{}^1P_1$ mixing of 45^o in the P-wave kaon sector; and assumed $f = 1$, $\kappa = 1$, $\delta = 1$ in order to *compare* with the widths Γ_2 of ref. [16]. Our optimal fit to ref. [16] gives widths Γ_1 (see §4).

A	B, C	L	Γ_1	Γ_2	A	B, C	L	Γ_1	Γ_2
$\pi 1^{-+}$	$b_1(1235)\pi$	S	100	100	$\phi 1^{-+}$	$K_1(1270)K$	D	90	80
		D	20	30		$K_1(1400)K$	S	200	250
	$f_1(1285)\pi$	S	30	30	$\pi 0^{+-}$	$a_1(1260)\pi$	P	600	800
		D	20	20		$h_1(1170)\pi$	P	100	100
$\omega 1^{-+}$	$a_1(1260)\pi$	S	90	100	$\omega 0^{+-}$	$b_1(1235)\pi$	P	250	250
		D	60	70	$\phi 0^{+-}$	$K_1(1270)K$	P	500	800
	$K_1(1400)K$	S	100	100		$K_1(1400)K$	P	70	50
$\pi 2^{+-}$	$a_2(1320)\pi$	P	350	450	$\omega 2^{+-}$	$b_1(1235)\pi$	P	350	500
	$a_1(1260)\pi$	P	100	100	$\phi 2^{+-}$	$K_2^*(1430)K$	P	300	250
	$h_1(1170)\pi$	P	125	150		$K_1(1400)K$	P	250	200

Table 3: Dominant widths in MeV for $\sqrt{\frac{1}{2}}(u\bar{u}-d\bar{d})$ *hybrid* $A \to BC$ for various J^{PC} in partial wave L. The quark model assignments for the mesons are those of the PDG tables [28]. All β's are rescaled from the ISGW / Merlin values by 5/4 to form "effective" β's consistent with that of $\beta = 0.4$. Hybrid masses before spin splitting are 2.0 GeV, except for 0^{+-} (2.3 GeV), 1^{+-} (2.15 GeV) and 2^{+-} (1.85 GeV), following ref. [29]. Final states containing π have $\tilde{\beta} = 0.36$ GeV, otherwise $\tilde{\beta} = 0.40$ GeV. For the hybrid we use $\beta_A = 0.27$ GeV. η indicates $\sqrt{\frac{1}{2}}(u\bar{u}+d\bar{d})$ at 550 MeV. The ${}^3P_1/\,{}^1P_1$-mixing is 34^o in the $L_B = 1$ kaon sector.

A	B,C	L	Γ	A	B,C	L	Γ	A	B,C	L	Γ
2^{-+}	$f_2(1270)\pi$	S	40	1^{+-}	$a_2(1320)\pi$	P	175	1^{-+}	$f_1(1285)\pi$	S	40
		D	20		$a_1(1260)\pi$	P	90			D	20
	$b_1(1235)\pi$	D	40		$h_1(1170)\pi$	P	175		$b_1(1235)\pi$	S	150
	$a_2(1320)\eta$	S	~ 40		$b_1(1235)\eta$	P	150			D	20
	$K_2^*(1430)K$	S	~ 30		$K_2^*(1430)K$	P	60		$a_1(1260)\eta$	S	50
2^{+-}	$a_2(1320)\pi$	P	200		$K_1(1270)K$	P	250		$K_1(1270)K$	S	20
	$a_1(1260)\pi$	P	70		$K_0^*(1430)K$	P	70		$K_1(1400)K$	S	~ 125
	$h_1(1170)\pi$	P	90	1^{++}	$f_2(1270)\pi$	P	175	0^{-+}	$f_2(1270)\pi$	D	20
	$b_1(1235)\eta$	P	~ 15		$f_1(1285)\pi$	P	150		$f_0(1300)\pi$	S	~ 150
0^{+-}	$a_1(1260)\pi$	P	700		$f_0(1300)\pi$	P	~ 20		$K_0^*(1430)K$	S	~ 200
	$h_1(1170)\pi$	P	125		$a_2(1320)\eta$	P	50	1^{--}	$a_2(1320)\pi$	D	50
	$b_1(1235)\eta$	P	80		$a_1(1260)\eta$	P	90		$a_1(1260)\pi$	S	150
	$K_1(1270)K$	P	600		$K_2^*(1430)K$	P	~ 20			D	20
	$K_1(1400)K$	P	150		$K_1(1270)K$	P	40		$K_1(1270)K$	S	40
					$K_1(1400)K$	P	~ 20		$K_1(1400)K$	S	~ 60

Table 4: As in table 3 but for initial hybrid $\sqrt{\frac{1}{2}}(u\bar{u}+d\bar{d})$.

A	B,C	L	Γ	A	B,C	L	Γ	A	B,C	L	Γ
2^{-+}	$a_2(1320)\pi$	S	125	2^{+-}	$b_1(1235)\pi$	P	250	1^{++}	$a_2(1320)\pi$	P	500
		D	60		$h_1(1170)\eta$	P	30		$a_1(1260)\pi$	P	450
	$f_2(1270)\eta$	S	~ 50	0^{+-}	$b_1(1235)\pi$	P	300		$f_2(1270)\eta$	P	70
	$K_2^*(1430)K$	S	~ 30		$h_1(1170)\eta$	P	90		$f_1(1285)\eta$	P	60
1^{+-}	$b_1(1235)\pi$	P	500		$K_1(1270)K$	P	600		$K_2^*(1430)K$	P	~ 20
	$h_1(1170)\eta$	P	175		$K_1(1400)K$	P	150		$K_1(1270)K$	P	40
	$K_2^*(1430)K$	P	60	1^{-+}	$a_1(1260)\pi$	S	100		$K_1(1400)K$	P	~ 20
	$K_1(1270)K$	P	250			D	70	0^{-+}	$a_2(1320)\pi$	D	60
	$K_0^*(1430)K$	P	70		$f_1(1285)\eta$	S	50		$f_0(1300)\eta$	S	~ 200
1^{--}	$K_1(1270)K$	S	40		$K_1(1270)K$	S	20		$K_0^*(1430)K$	S	~ 200
	$K_1(1400)K$	S	60		$K_1(1400)K$	S	~ 125				

Table 5: As in table 3 but for an initial $s\bar{s}$-hybrid. Hybrid masses before spin splitting are 2.15 GeV, except for 0^{+-} (2.25 GeV). Final states containing K have $\tilde{\beta} = 0.40$ GeV, otherwise $\tilde{\beta} = 0.44$ GeV. For the hybrid we use $\beta_A = 0.30$ GeV.

$s\bar{s}g$	B,C	L	Γ	$s\bar{s}g$	B,C	L	Γ	$s\bar{s}g$	B,C	L	Γ
2^{-+}	$K_2^*(1430)K$	S	100	1^{-+}	$K_1(1270)K$	S	40	0^{+-}	$K_1(1270)K$	P	400
	$K_1(1270)K$	D	20			D	60		$K_1(1400)K$	P	175
1^{+-}	$K_2^*(1430)K$	P	70		$K_1(1400)K$	S	250	0^{-+}	$K_2^*(1430)K$	D	20
	$K_1(1270)K$	P	250	2^{+-}	$K_2^*(1430)K$	P	90		$K_0^*(1430)K$	S	400
	$K_0^*(1430)K$	P	125		$K_1(1270)K$	P	30	1^{--}	$K_2^*(1430)K$	D	20
1^{++}	$K_2^*(1430)K$	P	125		$K_1(1400)K$	P	70		$K_1(1270)K$	S	60
	$K_1(1270)K$	P	70						$K_1(1400)K$	S	125
	$K_1(1400)K$	P	100								

Table 6: Partial wave amplitudes $\breve{M}_L(A \to BC)$ indicated in terms of the functions defined in eqn. 22 and named in accordance with partial waves S, P, D, F or G. We display various J^{PC} of the initial hybrid A decaying into pseudoscalar 0^{-+} (P) or vector 1^{--} (V) final mesons. Starred amplitudes vanish even with non-S.H.O. radial wave functions.

A	BC	$\breve{M}_L$	A	BC	$\breve{M}_L$	A	BC	$\breve{M}_L$
2^{-+}	VP	$-\sqrt{15}P/\sqrt{2}$	1^{--}	PP	$0 \times P^*$	1^{+-}	VP	$2\sqrt{3}S$
		F		VP	$3P$			$\sqrt{3}D/\sqrt{2}$
	VV	$3\sqrt{5}P$		VV	$3\sqrt{2}P$		VV	$0 \times S^*$
		F			$0 \times F^*$			$3D$
1^{-+}	PP	$3P$	2^{+-}	PP	$\sqrt{3}D$	0^{+-}	PP	$-\sqrt{6}S$
	VP	$3P/\sqrt{2}$		VP	$3D/\sqrt{2}$		VV	$-\sqrt{2}S$
	VV	$3\sqrt{2}P$		VV	$2\sqrt{10}S$			$-2D$
		F			$2\sqrt{2}D$	1^{++}	VP	$\sqrt{6}S$
0^{-+}	VP	$\sqrt{6}P$			G			$-\sqrt{3}D$
	VV	$0 \times P^*$					VV	$2\sqrt{3}S$
								$-\sqrt{6}D$

Table 7: Dominant widths in MeV for $\sqrt{\frac{1}{2}}(u\bar{u} - d\bar{d})$ *hybrid* $A \rightarrow BC$, where B and C are both L=0 quarkonia. $\Gamma = \Gamma_R \times (eqn.\,24)$. $\eta(\eta^{'})$ indicates $\sqrt{\frac{1}{2}}(u\bar{u} + d\bar{d})$ at 550 MeV (960 MeV) respectively. Starred Γ's tend to be ≤ 1 MeV, and are highly sensitive to model dependent assumptions about final state β's.

A	B,C	L	Γ_R	Γ	A	B,C	L	Γ_R	Γ	A	B,C	L	Γ_R	Γ
2^{-+}	$\rho\pi$	P	40	8	1^{+-}	$\omega\pi$	S	70	15	1^{--}	$\omega\pi$	P	40	8
	$\rho\eta$	P	15	4		K^*K	S	200	30		K^*K	P	30	4
	$\rho\eta^{'}$	P	8	2	1^{-+}	$\eta\pi$	P	40	$*$	1^{++}	$\rho\pi$	S	80	20
	K^*K	P	15	2		$\eta^{'}\pi$	P	40	$*$		$\rho\eta$	S	60	15
	$\rho\omega$	P	70	$*$		$\rho\pi$	P	40	8		$\rho\eta^{'}$	S	70	15
0^{-+}	$\rho\pi$	P	150	30		$\rho\eta$	P	20	4		K^*K	S	125	15
	$\rho\eta$	P	70	15		$\rho\eta^{'}$	P	9	2		$\rho\omega$	S	125	$*$
	$\rho\eta^{'}$	P	40	8		K^*K	P	15	2					
	K^*K	P	60	8		$\rho\omega$	P	50	$*$					

Table 8: As in table 7 but for initial hybrid $\sqrt{\frac{1}{2}}(u\bar{u} + d\bar{d})$.

A	B,C	L	Γ_R	Γ	A	B,C	L	Γ_R	Γ	A	B,C	L	Γ_R	Γ
1^{--}	$\rho\pi$	P	100	20	2^{-+}	K^*K	P	15	2	1^{+-}	$\rho\pi$	S	200	40
	$\omega\eta$	P	30	7	1^{-+}	$\eta^{'}\eta$	P	30	$*$		$\omega\eta$	S	100	20
	$\omega\eta^{'}$	P	15	3		K^*K	P	15	2		$\omega\eta^{'}$	S	150	30
	K^*K	P	30	4	1^{++}	K^*K	S	125	15		K^*K	S	200	30
2^{+-}	$\rho\pi$	D	5	1	0^{-+}	K^*K	P	60	8					

Table 9: As in table 7 but for initial hybrid $s\bar{s}$, and $\eta(\eta^{'})$ indicating $s\bar{s}$ at 550 MeV (960 MeV) respectively .

A	B,C	L	Γ_R	Γ	A	B,C	L	Γ_R	Γ	A	B,C	L	Γ_R	Γ
1^{--}	K^*K	P	90	15	1^{+-}	K^*K	S	150	20	1^{-+}	$\eta^{'}\eta$	P	70	$*$
	$\phi\eta$	P	60	8		$\phi\eta$	S	350	40		K^*K	P	50	6
	$\phi\eta^{'}$	P	15	2		$\phi\eta^{'}$	S	350	40	1^{++}	K^*K	S	80	10
2^{+-}	K^*K	D	6	1	0^{-+}	K^*K	P	175	30	2^{-+}	K^*K	P	40	6