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Study of N^* production —Version II—

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Abstract

Production of N^* in K^-p interations is studied in the reflectivity basis.

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1 Introduction

Consider a reaction

$$K^- p \to K^- N^{*+} \to K^- \pi^+ \pi^- p \tag{1}$$

in which an N^{*+} is produced via a Reggeon exchange in $K^- p$ interactions (see Fig. 1).

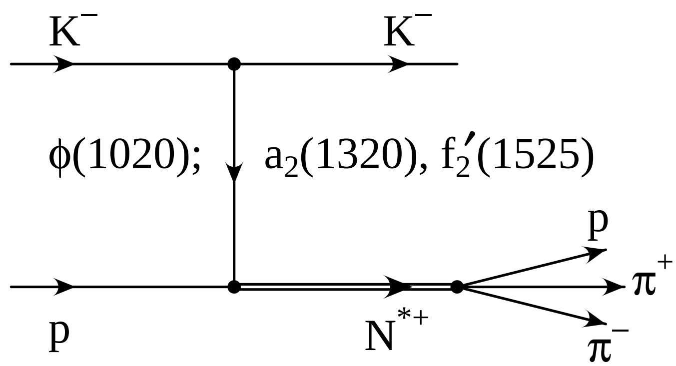

Figure 1: Resonance production via Reggeon exchange—an example.

The purpose of this note is to show how one may formulate N^* production amplitudes in the reflectivity basis. For a companion note on meson resonances produced at the upper vertex, the reader is referred to another note by the author[1].

2 Regge Amplitudes

Let $\alpha(t)$ be the Regge trajectory corresponding to an exchanged particle which may be $a_2^0(1320)$, $f_2'(1525)$ or $\phi(1020)$. Note that all of them belong to natural parity series. Let J and η be the spin and parity of the N^* and let $a = \{J, \eta\}$. Denoting the helicities by λ and μ for the initial and the final baryons, the Regge amplitude is, from Irving and Worden[2] [see their Equation (AA.10)],

$$A^a_{\lambda\mu}(s,t) \sim -\left(\frac{t'}{s'_0}\right)^{\frac{1}{2}n} \left[\frac{\mathrm{e}^{-i\pi\alpha(t)} + \mathcal{S}}{2\sin\pi\alpha(t)}\right] \beta_0\, \beta_{\lambda\mu} \left(\frac{s}{S_0}\right)^{\alpha(t)} \tag{2}$$

where

$$n = |\mu - \lambda| \tag{3}$$

Here $t' = (-t) - (-t)_{\min}$ and $\mathcal{S} = \pm 1$ is the signature of the Regge trajectory ($\mathcal{S} = (-1)^S$ where S is the spin of the particle responsible for the Regge trajectory). β_0 is the coupling constant (real) at the meson vertex and $\beta_{\lambda\mu}$, also real, is that at the baryon vertex. From parity conservation, one has

$$\beta_{\lambda\mu} = \nu_a\,(-)^{\mu-\lambda}\beta_{-\lambda\,-\mu} \tag{4}$$

where

$$\nu_a = \eta\,(-)^{J-\frac{1}{2}} \tag{5}$$

which may be termed the naturality of the N^*.

It is clear that the Regge amplitudes should obey the same relationship

$$A^a_{\lambda\mu}(s,t) = \nu_a\,(-)^{\mu-\lambda}A^a_{-\lambda\,-\mu}(s,t) \tag{6}$$

This result is quite general and does not depend on any particular form of the Regge amplitudes. The reader may derive this result by going to the center-of-mass frame for Reaction (1) and by considering a reflection operator through its reaction plane[3].

3 Angular Distributions

Finally one is ready to write down the angular distribution for the N^* decay in the reflectivity basis[4]. Let τ denote the 5-dimensional space representing the three-body decay of the N^*. Then the angular distribution is

$$I(\tau) = \frac{1}{2}\sum_{\lambda\lambda'}\left|\sum_{a\,\mu} A^a_{\lambda\mu}(s,t)\,F^a_{\mu\lambda'}(\tau)\right|^2 \tag{7}$$

where λ' is the helicity of the final nucleon produced in the N^* decay and F is a known function of τ describing the decay. The cross-section formula requires that one take the average of all possible initial spin states: the factor 1/2 in front takes care of this for the initial nucleon spin with helicities $\lambda = \pm\frac{1}{2}$. In terms of the density matrix, the angular distribution is

$$I(\tau) = \sum_{\lambda'}\sum_{\substack{a\,a'\\ \mu\mu'}} \rho^{a\,a'}_{\mu\mu'}\,F^a_{\mu\lambda'}(\tau)\,F^{a'\,*}_{\mu'\lambda'}(\tau) \tag{8}$$

where

$$\rho^{a\,a'}_{\mu\mu'} = \frac{1}{2}\sum_{\lambda} A^a_{\lambda\mu}(s,t)\,A^{a'\,*}_{\lambda\mu'}(s,t) \tag{9}$$

which satisfies, from parity conservation [see (6)],

$$\rho^{a\,a'}_{-\mu\,-\mu'} = \nu_a\nu_{a'}\,(-)^{\mu-\mu'}\,\rho^{a\,a'}_{\mu\mu'} \tag{10}$$

In the reflectivity basis, the angular distribution can be wrtiten

$$I(\tau) = \frac{1}{2} \sum_{\lambda\lambda'} \left| \sum_{a\,\mu\,\epsilon} {}^{\epsilon}\!A^{a}_{\lambda\mu}(s,t)\ {}^{\epsilon}\!F^{\,a}_{\mu\lambda'}(\tau) \right|^2 \tag{11}$$

where the new F's and A's are

$${}^{\epsilon}\!F^{\,a}_{\mu\lambda'}(\tau) = \theta(\mu) \left[F^{\,a}_{\mu\lambda'}(\tau) + \epsilon\, \nu_a\, (-)^{\mu-\frac{1}{2}}\, F^{\,a}_{-\mu\lambda'}(\tau) \right] \tag{12}$$

and

$${}^{\epsilon}\!A^{a}_{\lambda\mu}(s,t) = \theta(\mu) \left[A^{a}_{\lambda\mu}(s,t) + \epsilon^*\, \nu_a\, (-)^{\mu-\frac{1}{2}}\, A^{a}_{\lambda\,-\mu}(s,t) \right] \tag{13}$$

It is important to recognize that ϵ^* appears in A whereas ϵ is used in F: this can be traced to the fact that a 'ket' state for a is introduced in F, concomitant with a 'bra' state which is appropriate for defining A. One can show easily that, using the defintions (12) and (13), the distribution (11) is indeed the same as (7). From parity conservation [see (6)], one finds

$${}^{\epsilon}\!A^{a}_{-\lambda\mu}(s,t) = \epsilon^*\, (-)^{\lambda-\frac{1}{2}}\ {}^{\epsilon}\!A^{a}_{\lambda\mu}(s,t) \tag{14}$$

Consider now a factor contained in (11)

$$\sum_{\lambda} {}^{\epsilon}\!A^{a}_{\lambda\mu}(s,t)\ {}^{\epsilon'}\!A^{a'\,*}_{\lambda\mu'}(s,t) \tag{15}$$

The sum on λ can be broken up into two terms, one with $\lambda > 0$ and the other with $\lambda < 0$ ($\lambda = 0$ does not exist because the particle is a fermion). But the second term is proportional to the first from parity conservation (14), and one finds

$$(1 - \epsilon\epsilon') \sum_{\lambda>0} {}^{\epsilon}\!A^{a}_{\lambda\mu}(s,t)\ {}^{\epsilon'}\!A^{a'\,*}_{\lambda\mu'}(s,t) \tag{16}$$

which is zero unless $\epsilon = \epsilon'$.

In effect, the sum on ϵ in (11) can be taken out of the absolute square, to obtain

$$I(\tau) = \sum_{\substack{\lambda>0 \\ \epsilon\,\lambda'}} \left| \sum_{a\,\mu} {}^{\epsilon}\!A^{a}_{\lambda\mu}(s,t)\ {}^{\epsilon}\!F^{\,a}_{\mu\lambda'}(\tau) \right|^2 \tag{17}$$

Therefore, the distribution function in the reflectivity basis can be written

$$I(\tau) = \sum_{\epsilon\lambda'} \sum_{\substack{a\,a' \\ \mu\mu'}} {}^{\epsilon}\!\rho^{\,a\,a'}_{\mu\mu'}\ {}^{\epsilon}\!F^{\,a}_{\mu\lambda'}(\tau)\ {}^{\epsilon}\!F^{\,a'\,*}_{\mu'\lambda'}(\tau) \tag{18}$$

and now the density matrix in the reflectivity basis takes on the form

$$ {}^{\epsilon}\rho^{a\,a'}_{\mu\mu'} = \sum_{\lambda>0} {}^{\epsilon}A^{a}_{\lambda\mu}(s,t)\; {}^{\epsilon}A^{a'\,*}_{\lambda\mu'}(s,t) \tag{19} $$

As the initial baryon is a proton, its helicity is limited to $\lambda = \pm\frac{1}{2}$, and in this case one deduces an important result

$$ {}^{\epsilon}\rho^{a\,a'}_{\mu\mu'} = {}^{\epsilon}A^{a}_{\sigma\mu}(s,t)\; {}^{\epsilon}A^{a'\,*}_{\sigma\mu'}(s,t) \tag{20} $$

where $\sigma = +\frac{1}{2}$. One sees that this density matrix has rank one [instead of two—see (9)]. The distribution function now reads, from (17),

$$ I(\tau) = \sum_{\epsilon\,\lambda'} \left| \sum_{a\,\mu} {}^{\epsilon}A^{a}_{\sigma\mu}(s,t)\; {}^{\epsilon}F^{\,a}_{\mu\lambda'}(\tau) \right|^2 \tag{21} $$

It is clear that this formula is the most efficient one for applying to partial-wave analyses; it shows the A's to be determined from the data and it is expressed in terms of the known functions F of τ. Note that the distribution function is a sum of four non-interfering terms corresponding to $\epsilon = \pm i$ and $\lambda' = \pm\frac{1}{2}$. One can verify that, after some algebra, the distribution function (21) is indeed tha same as (7)—by using the definitions (12) and (13) and the parity-conservation relationship (6). More generally, one can take (17) and transform it into (7) through the same technique.

4 N^* decays

Consider a decay chain of an N^* of spin J resulting in an isobar b with spin S_0

$$ \begin{aligned} J &\to S_0 + S_3 \\ S_0 &\to S_1 + S_2 \end{aligned} \tag{22} $$

where S_1, S_2 and S_3 are the three particles in the final state, after a two-body decay of the isobar b. The parities and helicities will be denoted by η_i and λ_i for $i = 0,\ 1,\ 2$ and 3. One of the final particles S_1, S_2 and S_3 must be a nucleon and its helicity was denoted λ' in the previous section. Thus, if the isobar b is a Δ, then $S_0 = 3/2$, $S_1 = 1/2$ ($\lambda' = \lambda_1$), and S_2 and S_3 are the π's. On the other hand, if the isobar b happens to be a ρ, then $S_0 = 1$, $S_3 = 1/2$ ($\lambda' = \lambda_3$), and S_1 and S_2 are the π's. In any event, one has $S_2 = \lambda_2 = 0$ and $\eta_2 = -1$ for a pion. The 5-dimensional space τ can be represented by two angles (Θ, Φ) describing the orientation of the isobar in the N^* rest frame (the Jackson frame in which $\widehat{Z}$ is along the initial nucleon and $\widehat{Y}$ is along the production normal in the overall center-of-mass system), the isobar mass w, and two additional angles (θ, ϕ) which describe the S_1 orientation in the isobar helicity frame ($\widehat{z}$ is along the b direction in the b rest frame and $\widehat{y}$ is along $\widehat{Z} \times \widehat{z}$).

The decay amplitude is

$$F^a_{\mu\lambda_1\lambda_3}(\tau) = \left(\frac{2J+1}{4\pi}\right)\left(\frac{2S_0+1}{4\pi}\right)\sum_{\lambda_0} D^{J\,*}_{\mu\delta}(\Phi,\Theta,0)\; D^{S_0\,*}_{\lambda_0\lambda_1}(\phi,\theta,0)\; f^a_{\lambda_0\lambda_3}(w) f^b_{\lambda_1}(w) \tag{23}$$

where $\delta = \lambda_0 - \lambda_3$. Note that, for the problem under consideration, $\{\lambda_1, \lambda_3\} = \{\lambda'\}$, i.e. either λ_1 or λ_3 is zero for a pion (but not both). It is important to realize that the decay amplitude depends on *both* λ_1 and λ_3, one of which represents the final-state nucleon; consequently, there should be *no* summation on either λ_1 or λ_3 in the amplitude, and the summation is performed after taking the absolute square of the amplitudes [note the sum on λ' in (21)]. The helicity coupling amplitudes, which depend in general on the w, are related to the ℓs-coupling amplitudes through a standard prescription[3],

$$f^a_{\lambda_0\lambda_3}(w) = \sum_{L\Sigma}\left(\frac{2L+1}{2J+1}\right)(L0\,\Sigma\delta|J\delta)\,(S_0\lambda_0\;S_3\;-\lambda_3|\Sigma\delta)\,g^a_{L\Sigma}(w) \tag{24}$$

where Σ is the total intrinsic spin ($|S_0 - S_3| \le \Sigma \le S_0 + S_3$). The orbital angular momentum L is restricted to either even or odd, depending on the intrinsic parities involved, i.e. L is even (odd), if $\eta\eta_0\eta_3 = +1(-1)$. And one has

$$f^b_{\lambda_1}(w) = \left(\frac{2\ell+1}{2S_0+1}\right)(\ell 0\,S_1\lambda_1|S_0\lambda_1)\,g^b_\ell(w) \tag{25}$$

Note that there is no sum on ℓ, since S_1 is either 0 or 1/2, and that ℓ is even (odd), if $\eta_0\eta_1\eta_2 = +1(-1)$.

The ℓs-coupling amplitudes $g^a_{L\Sigma}$ and g^b_ℓ should be functions of w through the angular-momentum barrier factors and the isobar Breit-Wigner form, as follows:

$$\begin{aligned} g^a_{L\Sigma}(w) &= \left(\frac{p}{p_0}\right)^L x^a_{L\Sigma} \\ g^b_\ell(w) &= \Delta_\ell(w)\left(\frac{q}{q_0}\right)^\ell x^b_\ell \end{aligned} \tag{26}$$

where p is the break-up momentum for $J \to S_0 + S_3$ and q is that for $S_0 \to S_1 + S_2$. p_0 and q_0 are those evaulated at $w = w_0$, where w_0 is the isobar mass. $\Delta_\ell(w)$ is the Breit-Wigner function given by

$$\Delta_\ell(w) = \frac{w_0\Gamma_0}{w_0^2 - w^2 - iw_0\Gamma_\ell(w)} \tag{27}$$

where Γ_0 is the isobar width and

$$\Gamma_\ell(w) = \Gamma_0\frac{w_0}{w}\frac{q}{q_0}\left(\frac{q}{q_0}\right)^{2\ell} \tag{28}$$

In practice, the barrier factors, $(p/p_0)^L$ and $(q/q_0)^\ell$, are often replaced by Blatt-Weisskopf functions[5]. Finally, the variables x^a and x^b which appear in (26) are the parameters to be fitted, given a set of events of the type (1).

References

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