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Dead-time fractions for triggers

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Abstract

A formula for dead-time fraction is derived for a given set of input quantities: total beam flux, spill length, trigger rates and dead time for each trigger. The formula depends on one constant, the number of triggers per spill with an infinite flux.

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1 Introduction

It is a common problem in an experiment where one needs to know how to calculate the dead-time fraction given a total beam flux, spill length, trigger rates and dead time for each trigger. It is assumed that one is dealing with a set of triggers with a different dead time for each.

Intrinsic scale to the problem is shown to be the maximum possible number of triggers per spill corresponding to an infinite beam flux.

2 Calculation of dead-time fractions

The following notations are introduced to deal with this problem:

$$\begin{aligned} s &= \text{total flux} \simeq 1.5M \text{ per spill} \\ L &= \text{spill length} \simeq 800\, m\text{sec} \\ \Delta &= \text{total dead time in a spill} \\ n_i &= \text{number of events per spill for trigger } i \\ r_i &= \text{rate (per million) for trigger } i \\ d_i &= \text{dead time for trigger } i \simeq 1.0\, m\text{sec} \end{aligned} \tag{1}$$

where

$$n = \sum_i^K n_i \quad \text{and} \quad r = \sum_i^K r_i \tag{2}$$

Here K is the number of different triggers being taken per spill. It is assumed that all the quantities listed above are known, except Δ and n_i. Evidently, one has

$$n_i = r_i\, s\,(1-f) \quad \text{and} \quad n = r\, s\,(1-f) \tag{3}$$

where f is the dead-time fraction per spill:

$$f = \frac{\Delta}{L} \tag{4}$$

The total dead time Δ is given by

$$\Delta = \sum_i^K n_i\, d_i \tag{5}$$

Dividing both sides of (5) by L, one gets

$$f = \sum_i^K \frac{n_i}{m_i} \tag{6}$$

where m_i is the maximum possible number of events per spill for trigger i with an infinite flux (see below).

$$m_i = \frac{L}{d_i} \tag{7}$$

Inserting n_i of (3) into (6) and solving for f, one obtains

$$f = \frac{r\,s}{r\,s + \overline{m}} \tag{8}$$

where $\overline{m}$ is defined through

$$\sum_i^K \frac{r_i}{m_i} = \frac{r}{\overline{m}} \tag{9}$$

Eq. (8) is the key formula; it gives the dead-time fraction for a given r, s and $\overline{m}$. Note that the dead-time fraction f is small if r or s is sufficiently small, and it approaches unity, as r or s goes to infinity. The measure is provided by the size of $r\,s$ relative to that of $\overline{m}$, i.e. the intrinsic unit to this problem is given by $\overline{m}$.

It is instructive at this point to explore the meaning of $\overline{m}$. One finds, upon substituting (8) into (3),

$$n_i = \left(\frac{r_i\,s}{r\,s + \overline{m}}\right)\overline{m} \quad \text{and} \quad n = \left(\frac{r\,s}{r\,s + \overline{m}}\right)\overline{m} \tag{10}$$

This shows that the total number n of triggers per spill is given simply by $\overline{m}$, as the beam flux s approaches infinity. Similarly, the number n_i of events per spill for trigger i is given by $(r_i/r)\overline{m}$ for infinite s.

For example, consider the case of one trigger in the problem. Let the trigger rate r be 500 per million 'live' flux, while the total flux s is 1.5 million per spill. Then, one has $r\,s = 750$, and for $\overline{m} = 800$ one obtains $f = 48.4\%$. The actual triggers per spill is of course less than 500 per million; from (3), one finds $n = 387$.

Next consider the case of five triggers ($K = 5$). Let their trigger rates be 173, 72, 197, 83 and 198 per million, respectively. Assume for simplicity that the dead time is $1.0\,m$sec for all triggers, i.e. $d_i = 1.0\,m$sec. Then, one finds $r\,s = 1084.5$ and $\overline{m} = 800$, so that the overall dead-time fraction is $f = 57.6\%$. The expected numbers of triggers per spill are, in reality, 110, 46, 125, 53 and 126, respectively. Suppose now that the dead time for triggers 1 through 4 is $1.0\,m$sec, but the the trigger 5 has a dead time of $0.2\,m$sec. So one has $m_i = 800$ for $i = 1$ through 4 and $m_5 = 4000$; these are the maximum possible numbers of triggers for an infinite flux, if one had a single trigger in the problem. In the case of five triggers being taken simultaneously, one finds $\overline{m} = 1024.4$ from (9) and $r\,s = 1084.5$, so that $f = 51.4\%$. The expected numbers of triggers are now 126, 52, 144, 61 and 144, respectively. The maximum possible triggers per spill with an infinite flux is, from (10), 245, 102, 279, 118 and 281, respectively, and they add up to $\overline{m} = 1024.4$.