Blatt-Weisskopf centrifugal-barrier factors

as given by von Hippel and Quigg

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29-Oct-97

Barrier Factor

$$BF_L(p) = 1/(x^2 \mid h1_L(x)|^2)$$

In the notation that S.U. Chung uses in *Formulas for Partial-Wave Analysis Version II*
$F_L(p) = \mathrm{Sqrt}(BF_L(p))$ where $z = x^2$

$h1_L(x)$ are spherical Hankel functions of the first kind which are given by the generating function below:

In[103]:=

```
h1[n_,z_] := (-I)^(n+1)*Exp[I*z]/z*
    Sum[I^s/(s!(2*z)^s) (n +s)!/(n-s)! ,{s,0,n}]
```

The first 3 Hankel functions are given below:

In[104]:=

```
h1[0,z]
h1[1,z]
h1[2,z]
```

Out[104]=

$$\frac{-I\ E^{I\ z}}{z}$$

Out[105]=

$$-\left(\frac{E^{I\ z}\ (1 + \frac{I}{z})}{z}\right)$$

Out[106]=

$$\frac{I\ E^{I\ z}\ (1 - \frac{3}{z^2} + \frac{3\ I}{z})}{z}$$

Defining the Barrier Factors

```
BF[l_,q_] :=
 Simplify[1/(q^2 h1[l,q] ComplexExpand[Conjugate[h1[l,q]]])]
```

The first 10 unnormalized Barrier Factors

In[130]:=

```
BF[0,q]
BF[1,q]
BF[2,q]
BF[3,q]
BF[4,q]
BF[5,q]
BF[6,q]
BF[7,q]
BF[8,q]
BF[9,q]
```

Out[130]=

```
1
```

Out[131]=

$$\frac{q}{\frac{1}{q} + q}$$

Out[132]=

$$\frac{q^4}{9 + 3 q^2 + q^4}$$

Out[133]=

$$\frac{q^6}{225 + 45 q^2 + 6 q^4 + q^6}$$

Out[134]=

$$\frac{q^8}{11025 + 1575 q^2 + 135 q^4 + 10 q^6 + q^8}$$

Out[135]=

$$\frac{q^{10}}{893025 + 99225 q^2 + 6300 q^4 + 315 q^6 + 15 q^8 + q^{10}}$$

Out[136]=

$$q^{12} / (108056025 + 9823275 q^2 + 496125 q^4 + 18900 q^6 + 630 q^8 + 21 q^{10} + q^{12})$$

Out[137]=

$$q^{14} / (18261468225 + 1404728325 q^2 + 58939650 q^4 + 1819125 q^6 + 47250 q^8 + 1134 q^{10} + 28 q^{12} + q^{14})$$

Out[138]=

$$q^{16} / (4108830350625 + 273922023375 q^2 + 9833098275 q^4 + 255405150 q^6 + 5457375 q^8 + 103950 q^{10} + 1890 q^{12} + 36 q^{14} + q^{16})$$

Out[139]=

$$q^{18} / (1187451971330625 + 69850115960625\, q^{2} + 2191376187000\, q^{4} + 49165491375\, q^{6} + 893918025\, q^{8} + 14189175\, q^{10} + 207900\, q^{12} + 2970\, q^{14} + 45\, q^{16} + q^{18})$$

Normalization of Barrier Factors

Following S.U. Chung's normalization, the barrier factor are renormalized so that $B_L(p)$ = 1 for x = 1.

In[140]:=

```
1== A BF[0,1]
1== A BF[1,1]
1== A BF[2,1]
1== A BF[3,1]
1== A BF[4,1]
1== A BF[5,1]
1== A BF[6,1]
1== A BF[7,1]
1== A BF[8,1]
1== A BF[9,1]
```

Out[140]=

$$1 == A$$

Out[141]=

$$1 == \frac{A}{2}$$

Out[142]=

$$1 == \frac{A}{13}$$

Out[143]=

$$1 == \frac{A}{277}$$

Out[144]=

$$1 == \frac{A}{12746}$$

Out[145]=

$$1 == \frac{A}{998881}$$

Out[146]=

$$1 == \frac{A}{118394977}$$

Out[147]=

$$1 == \frac{A}{19727003738}$$

Out[148]=

$$1 == \frac{A}{4392846440677}$$

Out[149]=

$$1 == \frac{A}{1259543537287741}$$